ANALYSIS OF THE LIFT-OFF AND FURTHER MOTION OF A ROCKET

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ABSTRACT: Isaac Newton first proposed the three laws of motion in his Philosophiae Naturalis Principia Mathematica (The Mathematical Principles of Natural Philosophy), that was published in 1687. In doing so he laid the foundation of Newtonian mechanics. Lift-off of a rocket from a resting position from a launch pad could be said to represent all three Newton’s laws of motion. This article deals with the theoretical aspects of happenings at the atomic and molecular level - the interaction between nozzle atoms of a rocket and the high velocity exhaust gas molecules flowing through the nozzle - when a rocket lifts off and moves further. This interplay between nozzle atoms and exhaust gas molecules could be the driving force behind the motion of a rocket. In this paper theory and its explanation about interplay between nozzle atoms and exhaust gas molecules has been discussed in detail.

Key words: - Nozzle, Rocket, Lift-off, Thrust.

INTRODUCTION:
Newton’s third law of motion states that every force occurs as one member of an action/reaction pair of forces [1]. For each force exerted on one body, there is an equal, but oppositely directed, force on some other body interacting with it [2]. Lift-off and further motion a rocket is a prime example where Newton’s third law could be applied and that which could be analyzed in-depth further.

In a rocket using chemical propellants, fuel and oxidizer are burnt in the combustion chamber of that rocket engine resulting in high velocity, high temperature gases being ejected from the rocket nozzle. According to Newton’s third law, the continuous high velocity ejection of a stream of hot gases in one direction causes a steady motion of the rocket in the opposite direction because of the generation of necessary thrust.

At the macroscopic level, the force of exhaust gas in one direction and the thrust in the opposite direction form an action/reaction pair of forces – thrust being the reaction force. This is sufficient to explain the motion of rocket in usual sense. But, when we analyze deeper, then we realize that the actual interaction takes place between the exhaust gas molecules and the nozzle atoms at the microscopic level which is in fact responsible for generation of thrust. Fig. 1. shows the basic parts of a rocket.

Principal forces acting on a rocket:
There are four principal forces that act on a rocket during its lift-off – thrust, its weight under gravity and the two aerodynamic forces of drag and lift [5,6,7,8]. The vertical motion of a rocket is governed by the three forces of thrust, drag and weight [9]; the aerodynamic drag of the rocket body and the weight due to gravitational pull of the Earth are in the same downward direction and directly opposite to the upward direction of the thrust force[10]. The lift force could be
assumed to be negligible as a rocket is wingless and has a symmetrical design [11].
In many rockets designs, lift force generated due to steering mechanisms (aerodynamic fins, jet vanes, or gimbaled motor chamber) is used to stabilize and control the direction of flight like a steering force[12,13]. A rocket lifts off when the thrust generated because of the flow of the exhaust gas is more than the combination of drag and weight shown in fig.2[14,15].

**Nozzle of a rocket:**
A typical rocket engine consists of a combustion chamber, a nozzle, fuel injector and igniter. Fuel injector is used in case of a liquid propellant rocket engine.
For producing adequate thrust in a rocket engine, convergent-divergent nozzle (CD nozzle) also called as de Laval nozzle or supersonic nozzle is used, this is shown in fig.3 [16,17]. A CD nozzle has a converging section, a diverging section and a throat which is the area of minimum cross-section between the converging and diverging sections [17].
Throat increases the velocity of exhaust gas that flows from the combustion chamber into the diverging section of the nozzle as what could be explained by Bernoulli’s principle[18,19] and Venturi effect [20]. The desirable velocity of the exhaust through the throat of the nozzle is sonic (Mach 1) while the same through the skirt or divergent part is supersonic (Mach 2 or 3). A supersonic nozzle achieves a high degree of conversion of enthalpy to kinetic energy [21].
The combustion chamber of the rocket engine is specifically designed to contain high pressure and high temperature gases [22,23,24], because for a given thrust requirement, a higher pressure engine will be smaller in size [24]. When the throat size is such that the velocity of gas is sonic at the throat i.e. velocity is equal to that of the sound (343 m/s), then the flow is said to be choked; here the mass flow rate through the nozzle is maximum. In such a situation, further downstream, the flow is supersonic in the diverging section of the nozzle [25,26,27]. The exhaust gas leaves the nozzle exit plane at supersonic velocities[28]; it is a prerequisite for an efficient nozzle design that the flow within the entire nozzle exit cone be supersonic so that shock doesn’t develop in the exit cone, as that would result in dramatically less thrust [29].
The thrust force generated by a rocket engine could be explained using Newton’s third law of motion. Chemical propellants are burnt in the combustion chamber of a rocket engine. This generates a mixture of high velocity, high temperature gases that are ejected from the rocket nozzle. This continuous high velocity ejection of a stream of hot gases in one direction generates thrust in the opposite direction causing a steady motion of the rocket in that direction [3,4].
The resulting thrust is given by the following equation:

\[
\text{Thrust} = F = m \cdot V_e + (P_e - P_o) A_e \]

where,
\( m \) is mass flow rate of the propellant through the nozzle
\( V_e \) is exhaust velocity at nozzle exit
\( P_e \) is rocket gas pressure at nozzle exit
\( P_o \) is ambient or atmospheric pressure
\( A_e \) is exit area of nozzle (see Fig. 4)
The first term \( (m \cdot V_e) \) on the right-hand side of above equation is the momentum thrust or the jet thrust given by the product of mass flow rate of the propellant and its exhaust velocity relative to the vehicle. The second term \( (P_e - P_o) A_e \) on the right-hand side is the pressure thrust, consisting of the product of the cross-sectional area at the nozzle exit.
\(A_e\) and the difference between the gas pressure at the exit \(P_e\) and the ambient or the atmospheric pressure \(P_0\) \[^{32,33}\]. As could be observed from the above equation, thrust is dependent upon mass flow rate, exhaust velocity, exit area of the nozzle, rocket gas pressure and ambient or atmospheric pressure and it is independent of flight velocity of the vehicle. Rocket nozzles are designed such that their exhaust pressures are equal or slightly higher than the ambient pressure \[^{32}\]. Fig. 5 shows the throat area and exit area of a nozzle.

As altitude increases, atmospheric pressure decreases; so that thrust increases at higher altitudes. In the vacuum of space, \(P_0 = 0\) and the pressure thrust becomes maximum.

\[
F = m \cdot V_e + P_e \cdot A_e
\]  \[^{32}\]

Factors affecting mass flow rate of a propellant:

Mass flow rate \(m\) depends upon combustion chamber pressure, nozzle throat area, specific heat ratio of gases, molecular weight of mixture of combustion gases, and chamber temperature \[^{34,35}\].

Factors affecting exhaust gas velocity:

Velocity of exhaust gas at nozzle exit depends upon specific heat ratio of gases, molecular weight of mixture of combustion gases, chamber temperature, gas pressure at nozzle exit and chamber pressure \[^{36,37}\].

Theory of interaction between nozzle atoms and exhaust gas molecules during lift-off and thereafter:

We can better analyze the process of lift-off when we think about what happens at the atomic and molecular level.

A rocket nozzle is made up of a metallic alloy where atoms are arranged in a crystal lattice as shown in fig.6 \[^{38,39,40}\]. The bonding between metal atoms that results in such a structure could be explained by electron sea model (Fig. 7) as well as molecular orbital theory (band theory) \[^{41,42}\]. There are trillions and trillions of atoms arranged in a symmetrical, three dimensional and a circumferential fashion in a crystal lattice structure of the nozzle.

When the exhaust gas molecules move through the nozzle at a high speed, there is an interaction between the atoms of the nozzle and the molecules of the exhaust as a result of which the atoms of the nozzle are momentarily pushed down and then they immediately snap back upwards.

Now, there are rows and rows of atoms of the nozzle stacked one above the other symmetrically and circumferentially so that the combined force of interaction generated between these atoms and the fast moving exhaust molecules is very large. All the nozzle atoms are pushed downwards almost simultaneously and then they snap back upwards immediately; this cycle is repeated billions of times per second. (See Figures 8(a) to 8(d)) With each upward snapping motion of the atoms of the nozzle, the combined upward force is transmitted from the nozzle to the body of the rocket so that it is lifted upward and through a distance that could be measured in nanometers. Since these cycles are repeated billions of times per second, what we can perceive is a smooth upward lift-off of the rocket even though it takes place in nano steps.

Here we have to consider two types of distances between atoms of the nozzle- (1) stretch distance and (2) release distance. Due to downward flow of the exhaust gas molecules, the nozzle atoms are pushed downward maximally from their resting position. This distance between the resting...
and the maximum downward position of the nozzle atoms is the stretch distance. Suppose this is ‘x’. When the nozzle atoms snap back upwards, they cross the resting position and move further upward. This distance from the maximal downward position to maximal upward position is the release distance. Suppose this is ‘y’. Then we can say that release distance must be greater than the stretch distance if the rocket were to lift off i.e. y>x. (see Fig. 9)

This tiny difference between y and x is the lift-off cycle distance (LCD). Suppose this is equal to ‘z’. Then, \( z = y - x \). Here we can have following scenarios-

(a) \( y = x \), here, the LCD is zero so that the rocket stays stationary.
(b) \( y > x \), the LCD is greater than zero, so that the rocket starts its upward journey.

Let’s try to understand this using an example where \( y>x \). Suppose the lift-off velocity of the rocket in 1st second is 1m/s and its acceleration is 1m/s². (Let’s try to keep these numbers small for the ease of explanation.) Also, suppose that the stretch distance \( (x) \) is equal to 100 picometer and the release distance \( (y) \) is equal to 200 picometer, then \( z \) will be equal to 100 picometer or 0.1 nanometer. This means that the actual upward displacement of the nozzle atoms is equal to 0.1 nanometer or 100 picometer during each cycle; the body of the rocket is lifted up through the same distance per cycle. The new topmost positions of the nozzle atoms become their new resting positions at the beginning of each cycle, so that at the end of each cycle the rocket moves up by a distance of 0.1 nm. Now to cover a distance of 1 meter in 1 second, the nozzle atoms have to oscillate upward and downward 10 billions of times per second. This can be calculated as follows:

In one cycle, the distance covered by the nozzle atoms is 0.1 nm.

Suppose, it takes \( n \) number of cycles to cover a distance of 1 m.

Then, \( n/1m = 1/0.1nm \)

Therefore, \( n = (1/0.1nm) \times 1m \)

\( = (1/0.1nm) \times 10^9 \) nm

(since 1m = \( 10^9 \) nm)

\( n = 10^{10} \)

\( n = 10 \) billions

Likewise, to cover a distance of 2 meters in one second, the nozzle atoms have to oscillate 20 billions of times per second and so forth. To generate such a high frequency of oscillation, the exhaust gas must have a very high velocity while the high temperature further facilitates increased vibratory motion of particles. Also, higher the mass flow rate of propellants, larger is the number of gas molecules available to interact with greater number of nozzle atoms per unit time.

Thus, a rocket will continue to accelerate after lift-off as long as the exhaust velocity is above a certain threshold and as long as the release distance is greater than the stretch distance.

Motions of jet planes (commercial as well as fighter) could be explained in similar way, the only difference being the horizontal motion here instead of a vertical one.

**Effect of high temperature of gases:**

When atoms are exposed to high temperature, they gain high kinetic energy and their excursions from the resting position also called as the vibration amplitude can get significantly large. This kind of atom motion is called as thermal motion or thermal vibration[43]. Exhaust gases that have high temperature further augment vibratory motions of atoms, thus contributing to overall thrust.
Practical applications of the above theory in modifying the nozzle design: If the area of interaction between the nozzle atoms and the fast moving exhaust gas molecules is increased then, it will result in recruitment of large number of nozzle atoms. More the number of nozzle atoms involved, larger will be the force transmitted to the rocket body and larger will be the resulting upward displacement per cycle. This could be achieved by employing the following modifications in nozzle design:

(1) **Multiple exhaust outlets:** The exhaust nozzle of a rocket has a single outlet. In this scenario, the high velocity gas flowing through the centre of the throat and the diverging section of the nozzle doesn’t take part in any interaction with the nozzle atoms and hence is wasted. On the other hand, if a nozzle of the same size has multiple symmetrical outlets, then the surface area for the interaction between the exiting gas and the nozzle atoms is greatly increased. In this case, more thrust will be generated for less amount of fuel consumed. (see Figures 10(a) and 10(b)).

(2) **Extended throat:** The throat is the narrowest part of the nozzle with minimum length. If the length of throat is extended or increased till permissible limits (see Fig. 11), then as explained above, the area for interaction between the throat atoms and the exhaust gas molecules will increase resulting in more thrust. Also, as the throat walls will be vertical, it will result in pronounced movement of throat atoms in vertical axis as compared to atoms of the diverging section that move along an axis that is inclined by certain angle to the vertical.

The boundary layer thickness is minimum at the throat\(^{[44]}\), hence conditions are most favorable here for the interaction between nozzle atoms and gas molecules. Also, maximum gas flow rate per unit area occurs at the throat.\(^{[45]}\) Keeping these factors in mind, these modifications could definitely prove to be impactful in maximizing the thrust.

These modifications could be put to use in space rockets as well as in commercial aeroplanes and fighter jets. In case of the space rockets, they will reduce the amount of fuel needed and thus the overall weight of the rocket, while in cases of commercial aeroplanes and fighter jets, they will increase the flying ranges for the same quantity of fuel.

**CONCLUSION:**

There are many observations and factors that consolidate this theory.

(a) Thrust is proportional to mass flow rate of propellant and the exhaust velocity for obvious reasons. More the mass flow rate and exhaust velocity, more profound is the interaction between nozzle atoms and gas molecules, thus generating more thrust.

(b) Thrust is independent of flight velocity\(^{[46]}\)

Exhaust velocity is inversely proportional to molecular weight of exhaust gas. Smaller the average molecular size of gas, better will be the interaction between them and the nozzle atoms.

(c) Exhaust velocity is directly proportional to temperature of gas; more the exhaust velocity, higher will be the stretch and release distances that will translate into higher thrust.

(d) Higher the temperature of exhaust gas, more will be the vibration amplitude of the nozzle atoms due to heat exchange and higher will be the stretch and release distances and thrust.

Taking into consideration the above factors, this theory could be put to test empirically.
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Fig. 1 Basic parts of a rocket

Fig. 2 Forces acting on a rocket during its launch
**Fig. 3 De Laval nozzle (CD nozzle)**

![Diagram of De Laval nozzle](image)

**Fig. 4 Rocket Thrust Equation**

![Diagram of Rocket Thrust Equation](image)

\[
\text{Thrust} = F = \dot{m} V_e + (P_e - P_0) A_e
\]

- \(\dot{m}\) = mass flow rate
- \(V_e\) = Exhaust velocity at nozzle exit
- \(P_e\) = Pressure at nozzle exit
- \(P_0\) = Ambient pressure
- \(A_e\) = Nozzle exit area
Fig. 5 Throat area and exit area of a nozzle

![Diagram of a nozzle with labeled parts: Combustion chamber, Throat area of nozzle, Diverging section of nozzle, Exit area of nozzle.]

Fig. 6 Three dimensional lattice arrangement of nozzle atoms

![Diagram of a three-dimensional lattice arrangement of nozzle atoms with labels: Interatomic distance, Nozzle atoms.]

Fig. 7 Metallic Bonding: The Electron Sea Model

Fig. 8(a) Equidistant nozzle atoms (in 2-D) in resting position

Fig. 8(b) Exhaust gas molecules at high speeds and high temperatures interact with nozzle atoms

Fig. 8(c) Increase in interatomic distances between nozzle atoms as they are pushed downwards by high speed exhaust gas molecules that are continuously flowing downwards

Fig. 8(d) Nozzle atoms snap back upwards, thus crossing their resting positions. The combined upward force so generated is transmitted from the nozzle to the body of the rocket. Rocket body gets lifted in upward direction through a distance that could be measured in nanometers per cycle. (As the atoms cross their resting positions, the interatomic distances between them are restored.)
Fig. 9(a), (b) & (c) Resting position, maximal downward and maximal upward position respectively, of a nozzle atom.

Fig. 10(a) A nozzle with single exhaust outlet as seen from below.

Fig. 10(b) A nozzle with multiple exhaust outlets. If coupled with an extended nozzle throat, this type of modification could generate more thrust with less amount of fuel consumed.

Fig. 11 De Laval nozzle with an extended throat.