International Journal of Researches In Biosciences, Agriculture & Technology
May 2014
Issue-2, Volume-II
ISSN No. (Online) 2347-517X

BIANCHI TYPE I HYPERBOLIC MODEL DARK ENERGY IN BIMETRIC THEORY OF GRAVITATION

Vijay Lepse
Science College, Pauni-441910, India

Abstract:
In this paper, we presented the solutions of Bianchi Type-I space -time with dark energy (quintessence and Chaplygin gas) by solving them Rosen's field equation in Bimetric theory of gravitation. The hyperbolic geometric view point of the models will be helpful to the people who use observational data to search for such types of geometry.
Keywords : Bianchi type-I, biometric, gravitational theory, cosmology, hyperbolic geometric, isotropize geometry
MSC 2010 No: 83D05;83F05

1. Introduction

Several theories of gravitation have been proposed as alternative to Einstein's theory of gravitation in order to explain the cosmic acceleration and existence of dark energy and dark matter in the universe. One among them is Rosen's Bimetric theory of gravitation. The latest discovery of modern cosmology is that the current universe is expanding and accelerating. The data base like Cosmic Microwave Background Radiation (CMBR), the experiment of Tegmark et al. (2004) and such as Type Ia supernova (S Ne Ia) observational data of Eisenstein et al.(2005), Astier et al. (2006) and Spergel et al. (2007) indicate that the universe containing only 4% ordinary baryonic matter, 23% dark matter and remaining 73% dark energy. The dark matter is unknown form of matter which has clustering properties of ordinary matter and has not been yet detected. The dark energy is the term represents for an unknown form of energy which has not also been detected. The equation of state parameter $\omega(t)$ has an important role in dark energy models. The constant values of $\omega = -1, 0, +1/3$ and $+1$ represents vacuum fluid, dust, radiation and stiff dominated universe. The variable $\omega(t)$ of time or red shift is considered and the quintessence model, $\omega > -1$ (explanation of observations of accelerating universe) involving scalar field and phantom model $\omega < -1$ (expansion of universe increases to infinite degree in finite time) give rise time dependant parameter $\omega(t)$ discussed by Turner et al. (1997), Jimenez (2003), Das et al. (2005). Various forms of parameter $\omega(t)$ have been used for dark energy models and a binary mixture of perfect fluid and dark energy have been considered by many researchers like Caldwell et al. (1998), Liddle et al. (1999), Steinhardt et al. (1999), Saha (2005, 2006), Rahaman et al. (2006, 2009), Mukhopadhyay et al. (2008), Singh et al. (2009) Ray et al. (2010), Akarsu et al. (2010), Sharif et al. (2010), Yadav et al. (2011) and Pradhan et al. (2011) to investigate the cosmological models of the universe and deduced the different geometrical and physical aspects of the models.

2 Metric and Field Equations

We consider the Bianchi Type I metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \tag{2.1}$$

where A, B and C are the functions of t only.

The flat metric corresponding to metric (2.1) is

$$d\eta^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{2.2}$$

The energy momentum tensor T_i^j of the source with perfect fluid and dark energy is given by

$$T_i^j = (\rho + p)v_i v^j + p\delta_i^j \tag{2.3}$$ with

$$v_i v^j = -1 \tag{2.4}$$

Here p and ρ are the pressure and energy density of matter respectively and v^i is the flow vector.

The quantity θ is the scalar of expansion which is given by

$$\theta = v^{i}{}_{|\,i} \tag{2.5}$$

We assume the coordinates to be co-moving, so that

$$v^1 = v^2 = v^3 = 0, \quad v^4 = -1$$

The equation (3.2.3) of energy momentum tensor yield its components as

$$T_1^1 = T_2^2 = T_3^3 = p \quad \text{and} \quad T_4^4 = -\rho \tag{2.6}$$

Our aim is to solve Rosen's field equations (1.1) and (1.2) for the metric (2.1) and (2.2). For this purpose, first we are going to calculate the components of Rosen's Ricci tensor N_i^j from equations (1.2) for $i, j = 1,2,3,4$. For $i = j = 1$, the component N_1^1 of Rosen's Ricci tensor N_i^j, from equation (1.2) is

$$N_1^1 = \frac{1}{2}\gamma^{\alpha\beta}\left(g^{s1} g_{s1|\alpha}\right)_{|\beta}$$

After solving , we get the following equations

$$-\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2} - \frac{\dot{C}^2}{C^2} = -16\pi ABC\, p \tag{2.7}$$

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} - \frac{\dot{C}^2}{C^2} = -16\pi ABC\, p \tag{2.8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} = -16\pi ABC\, p \tag{2.9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2} - \frac{\dot{C}^2}{C^2} = 16\pi ABC\, \rho \tag{2.10}$$

From equations (2.7) and (2.8), we write

$$A = B$$

and from equations (2.7) and (2.9), we get

$$A = C$$

Thus we have

$$A = B = C \tag{2.11}$$

Adding the equations (2.7) and (2.10), we get

$$\frac{\dot{A}^2}{A^2} - \frac{\ddot{A}}{A} = -8\pi ABC(p + \rho) \tag{2.12}$$

The volume V is the function of t and it is defined as

$$V = ABC \tag{2.13}$$

From the above equations we write

$$V = A^3 \quad i.e., \quad A = B = C = V^{1/3} \tag{2.14}$$

By straightforward calculations, we get

$$\frac{\ddot{V}}{V} - \frac{\dot{V}^2}{V^2} = 24\pi V(p + \rho) \tag{2.15}$$

We are going to solve this differential equation (3.2.17) for the Volume V and then using the equation (3.2.16), we are calculating the scale factors A, B and C and then we plan to study three different models in related with quintessence and chaplygin gas dark energy

3 Model with dark energy Quintessence model

Let us consider the model with dark energy in which the dark energy is given by quintessence which obeys the equation of state

$$p_q = \omega_q \rho_q, \quad -1 \le \omega_q \le 0 \tag{3.1}$$

in which ω_q is equation of state parameter corresponds to quintessence model.

The conservation law of energy momentum tensor, equation in the form of dark energy given by quintessence, can be written as

$$\rho_q = \frac{c_4}{V^{(1+\omega_q)}}$$

where c_4 is an integration constant.

Using the value of energy density ρ_q, we write the pressure p_q as

$$p_q = \frac{\omega_q c_4}{V^{(1+\gamma)}}$$

Thus we have the pressure p_q and energy density ρ_q of the perfect fluid as

$$p_q = \frac{\omega_q c_4}{V^{(1+\omega q)}}, \qquad \rho_q = \frac{c_4}{V^{(1+\omega q)}} \tag{3.2}$$

where c_4 is the constant of integration.

$$\int \frac{dV}{\sqrt{c_5 V^2 - 48\pi c_4 (1 + 1/\omega_q) V^{2-\omega q}}} = t + c_6 \tag{3.3}$$

where c_5 and c_6 are constants of integration. In particular, $c_6 = 0,$ we write

$$\int \frac{dV}{\sqrt{c_5 V^2 - 48\pi c_4 (1 + 1/\omega_q) V^{2-\omega q}}} = t \tag{3.4}$$

we discuss the solution with the particular cases of $\omega_q = 0, -\frac{1}{3}$ and -1.

Case (i) For $\omega_q = 0$ (even though it is a case of dusty universe, it is considered in quintessence equation of state), the integral equation has no solution and hence the model does not exist.

Case (ii) For $\omega_q = -\frac{1}{3}$. The integral equation has a solution (there is no need to repeat the calculation part, as it is similar to the case of perfect fluid for $\gamma = \frac{1}{3}$)

$$V = \frac{1}{(\sinh(mt + n))^6} \tag{3.5}$$

where m and n are non zero positive constants given by $m=\dfrac{\sqrt{96\pi c_4}}{6}$ and $n=m\,k$, in which k is constant. From equations, we have the scale factors

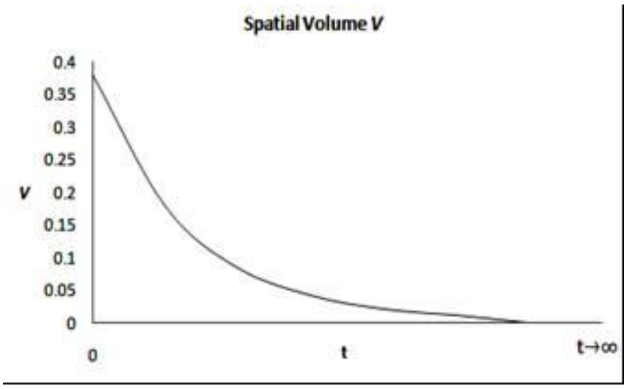
$$A=B=C=\ V^{1/3}=\frac{1}{(\sinh(mt+n))^2} \tag{3.6}$$

Thus the required metric is

$$ds^2=-dt^2+\frac{1}{(\sinh(mt+n))^4}\ (dx^2+dy^2+dz^2) \tag{3.7}$$

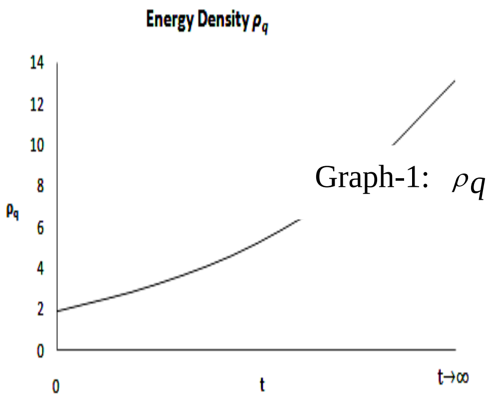
This is Bianchi type I cosmological model with quintessence dark energy in Bimetric theory of gravitation for $\omega_q=-1/3,$ which represents dark energy star.

This model has hyperbolic geometry and its volume V and scale factors $A,\ B$ and C are in inverse of hyperbolic sine functions. From the Graph-13, it is observed that, at $t=0,$ the volume of the model attain the maximum value and it is gradually decreasing as t increasing and approaches to zero value, when $t\rightarrow\infty.$ This shows that the model starts with maximum volume and the volume slowing down and approaches to zero at later stage.

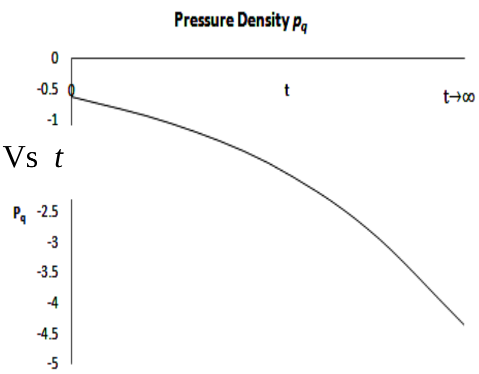


From equations the pressure p_q and the energy density ρ_q, corresponding to quintessence model are

$$\rho_q=-3p_q=c_4\ (\sinh(mt+n))^4 \tag{3.8}$$



Graph-1: ρ_q Vs t

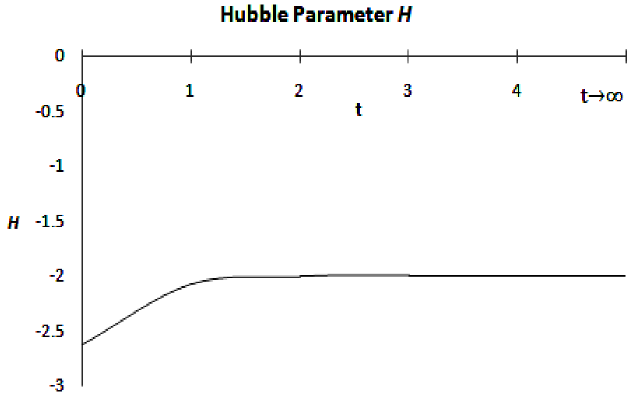


The natu Graph-2: ρ_q Vs t raph of hyperbolic sin Graph-3: p_q Vs t ity attain the non zero positive value and goes on increasing, as time t increasing and reaches to infinity, when

$t \to \infty$ as shown in Graph-2. This shows that in the beginning, the model has matter and density of the matter goes on increasing with time t and it is infinity at later stage. It is shown in Graph-3 that the nature of the pressure p_q in the model is negative and decreasing in nature and tends to minus infinity at later stage.

The Hubble parameter H and its directional's

$$H = H_1 = H_2 = H_3 = -\ 2m\coth(mt+n) \qquad (3.4.9)$$



Graph-4: H Vs t

Graph-16 shows the nature of th ; directional's H_1, H_2 and H_3 and its graph is in inverse of hyperbolic tangent function and the graph lies in IV[th] quadrant. This shows that there is a negative rate of expansion in the model, which indicate contracting model.

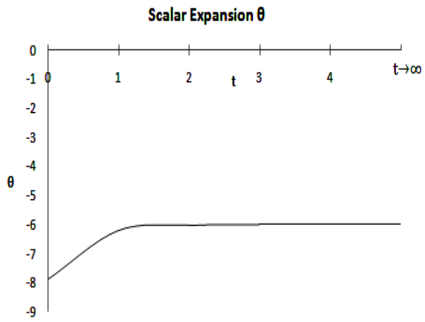
In the quintessence dark energy model for $\omega_q = -\dfrac{1}{3}$, the physical parameters θ, A, σ and q have been calculated as

$$\theta = -6m\coth(mt+n) \qquad (3.10)$$

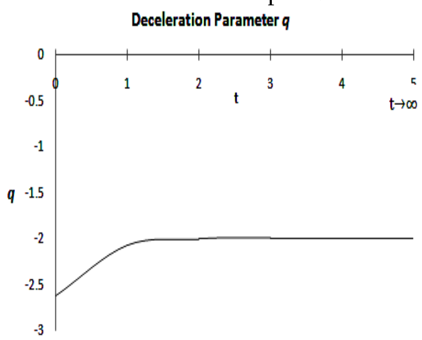
$$A = \sigma = 0 \qquad (3.11)$$

$$q = -\frac{1}{2}\operatorname{sech}^2(mt+n) - 1. \qquad (3.12)$$

The scalar expansion θ in the model obeys the same nature as that of Hubble parameter and it is always negative supporting the contraction of the model as shown in Graph-5.



Graph-5: θ Vs t



Graph-6: q Vs t

The de ays negative and i Thus this quintessence dark energy model is represents dark energy star and it is contracting starting with maximum volume.

Case (iii) For $\omega_q = -1$ the integral equation becomes $\int \frac{dV}{\sqrt{c_5}\, V} = t$

which has a solution

$$V = e^{\sqrt{c_5}\, t/3}$$

The scale factors are

$$A = B = C = e^{at}$$

where $a = \sqrt{c_5}/3$ is a positive constant.

Thus, the required metric is

$$ds^2 = -dt^2 + e^{2at}(dx^2 + dy^2 + dz^2)$$

This is the Bianchi type I cosmological model with dark energy (quintessence) in Bimetric theory of gravitation for $\omega_q = -1$ which has volumetric exponential expansion. The volume V and scale factors A, B and C are in exponential law. At $t = 0$, they admit value one and increasing exponential, as t increasing and reaches to infinity, when $t \to \infty$. This shows that the model starts evolving with metric of special relativity, with non zero volume and non zero scale factors and the volume and the scale factors are exponential increasing as t increases and admit infinite values at later stage.

The Hubble parameter H from equation and its directional's H_1, H_2, H_3 are

$$H = H_1 = H_2 = H_3 = a.$$

The physical parameters θ, A, σ and q admits the values

$$\theta = 3a, \quad A = \sigma = 0, \quad q = -1.$$

It is seen that the model has constant rate of expansion forever. The density ρ_q and pressure p_q of the matter for $\omega_q = -1$ is constant right from the beginning. The scalar expansion θ admits constant value which supports the uniform expansion. Anisotropic parameter A and shear σ admit zero values shows that the model is isotropize in all directions without shear. The deceleration parameter q is $q = -1$ shows model has accelerating phase forever and there is no decelerating phase.

4 Conclusion

Three different Bianchi type I cosmological models with quintessence dark energy, in Bimetric theory of gravitation have been deduced.. The dark energy quintessence model, for $\omega_q = -1/3$ is in contracting in nature and for $\omega_q = -1$, the model is expanding.. The hyperbolic geometric view point of the models will helpful to the peoples of observational data to search such type of geometry.

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