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Bianchi Type-I Perfect Fluid Model in Bimetric Theory of Gravitation

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Abstract

Bianchi type-I cosmological model as related to perfect fluidinbimetric theory of gravitation have been deduced. The perfect fluid model has hyperbolic geometry and all its physical parameters are also hyperbolic in nature and therefore they have been studied from hyperbolic geometric view point. All these models are isotropize and shear-less. Other geometrical and physical behaviors of the models have also been studied.

Keywords: Bianchi type-I; bimetric gravitational theory; cosmology; hyperbolic geometric; isotropize;geometry

1. Introduction

Rosen's (1973, 1975) bimetric theory of gravitation is one of the alternatives to general relativity and it is free from singularities appearing in the big-bang of cosmological models and it obeys the principle of covariance and equivalence of the general relativity. Therefore, the people are interested in investigating the cosmological models of the universe in bimetric theory of gravitation based on two matrices; one is Riemannian metric which described the geometry of curved space time, and the second is flat metric which refers to the geometry of the empty universe (no matter but gravitation is there) and described the initial forces.

The Rosen's field equations in bimetric theory of gravitation are

$$N_i^j-\frac{1}{2}N\,\delta_i^j=-T_i^j, \tag{1}$$

where

$$N_i^j=\frac{1}{2}\gamma^{pq}\left(g^{sj}g_{si|p}\right)_{|q},$$

$N=g^{ij}N_{ij}$ is the Rosen scalar. The vertical bar (|) stands for γ-covariant differentiation where $g=\det(g_{ij})$ and $\gamma=\det(\gamma_{ij})$. Many researchers have developed the theory and investigated many cosmological models of the universe in bimetric theory of gravitation and in general relativity, and studied their behavior geometrically and physically [Karade (1980), Israelit (1981), Reddy et al. (1989, 1998), Mohanty et al. (2002), Bali (2003a, 2003b, 2005, 2006, 2007), Katore (2006), Khadekar (2007), Borkar (2010a, 2013, 2014a, 2014b), Gaikwad (2011)]. Athough the non – existence of Bianchi types I, III, V and VI_0 cosmological models with perfect fluid in the Rosen's bimetric theory of gravitation have been shown by Reddy et al. (1989, 1998), Mohanty et al. (2002) and Borkar et al. (2010b) have deduced the existence of Bianchi type I magnetized cosmological model in Bimetric theory of gravitation and studied its geometrical and physical properties.

2. Solutions of Rosen's fieldequations

We consider the Bianchi Type I metric [Borkar et al. (2010b)] in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2, \tag{2}$$

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where A, B and C are functions of t only.

The flat metric corresponding to metric (2) is

$$d\eta^2 = -dt^2 + dx^2 + dy^2 + dz^2. \tag{3}$$

The energy momentum tensor T_i^j of the source (Sharif et al. (2013)) is given by

$$T_i^j = (\rho+p)v_i v^j + p\delta_i^j, \tag{4}$$

with

$$v_i v^i = -1. \tag{5}$$

Here, ρ and p are the energy density and pressure of perfect fluid, respectively and v^i is the flow vector. The quantity θ is the scalar of expansion which is given by

$$\theta = v^i_{\ ;i} \tag{6}$$

We assume the coordinates to be co-moving, so that

$$v^1 = v^2 = v^3 = 0, \qquad v^4 = -1.$$

Equation (4) of energy momentum tensor yield

$$T_1^1 = T_2^2 = T_3^3 = p \quad \text{and} \quad T_4^4 = -\rho. \tag{7}$$

The pressure p and the density ρ are related by an equation of state $p = \gamma\rho$, $0 \le \gamma \le 1$.
The Rosen's field Equations (1) for the metric (2) and (3) with the help of (7) give the differential equations

$$-\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2} - \frac{\dot{C}^2}{C^2} = -16\pi \quad ABCp, \tag{8}$$

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} - \frac{\dot{C}^2}{C^2} = -16\pi \quad ABCp, \tag{9}$$

From Equations (8) – (10), we write

$$A = B = C. \tag{12}$$

Also, from Equations (8) and (11), we get

$$\frac{\dot{A}^2}{A^2} - \frac{\ddot{A}}{A} = -8\pi ABC\ (p+\rho). \tag{13}$$

The volume V is a function of t and it is

$$V = ABC. \tag{14}$$

From Equation (12), we write

$$V = A^3, \qquad i.e., \qquad A = B = C = V^{\frac{1}{3}} \tag{15}$$

From Equations (13) - (15), we get

$$\frac{\ddot{V}}{V} \quad \frac{\dot{V}^2}{V^2} = 24\pi V\ (p+\rho). \tag{16}$$

3. The perfect fluid model with physicalsignificance

The conservation law for the energy-momentum tensor from Equation (4), we write

$$\dot{\rho} = -\frac{\dot{V}}{V}\ (p+\rho). \tag{17}$$

The perfect fluid obeys the equation of state

$$p_{PF} = \gamma \quad \rho_{PF}, \quad 0 \le \gamma \le 1. \tag{18}$$

Here, $\gamma = 0$ (Dust Universe), $\gamma = 1/3$ (Radiation Universe), $\gamma = \left(\frac{1}{3}, 1\right)$ (HardUniverse)

and$\gamma = 1$ (Zel'dovich Universe or stiff matter).

In a co-moving coordinate, the conservation law of energy momentum tensor (17), for the perfect fluid and dark energy is

$$\dot{\rho}_{DE} + \dot{\rho}_{PF} = -\frac{\dot{V}}{V}(\rho_{DE} + \rho_{PF} + p_{DE} + p_{PF}), \tag{19}$$

from which we have

$$\dot{\rho}_{DE} + \frac{\dot{V}}{V}(\rho_{DE} + p_{DE}) = 0, \tag{20}$$

and

$$\dot{\rho}_{PF} + \frac{\dot{V}}{V}(\rho_{PF} + p_{PF}) = 0. \tag{21}$$

From Equations (18) and (21), we write

$$\rho_{PF} = \frac{c_1}{V^{(1+\gamma)}}, \quad p_{PF} = \frac{\gamma c_1}{V^{(1+\gamma)}}, \tag{22}$$

where c_1 is an integration constant. Using Equation (22), Equation (16)infers

$$\dot{V} = \pm\sqrt{c_2 V^2 - 48\pi \frac{(1+\gamma)}{\gamma} c_1 V^{2-\gamma}}, \tag{23}$$

where c_2 is a constant of integration.
This Equation (23) has a solution

$$\int \frac{dV}{\sqrt{c_2 V^2 - 48\pi\left(1 + \frac{1}{\gamma}\right) c_1 V^{2-\gamma}}} = t + t_0, \tag{24}$$

where t_0 is constant of integration and is taken to be zero. So

$$\int \frac{dV}{\sqrt{c_2 V^2 - 48\pi\left(1 + \frac{1}{\gamma}\right) c_1 V^{2-\gamma}}} = t. \tag{25}$$

We study the model in view of the values of γ.

Case(i) For $\gamma = 0$, the integral Equation (25) has a singularity in dust regime. The model does not predict or does not allow a phase in which the universe is dominated by dust.

Case (ii) For $\gamma = 1/3$, the solution of integral Equation (25) is

$$V = \left[c_3 / \sqrt{c_2} \cosh\left(\left(\sqrt{c_2}/6\right)t\right)\right]^6, \tag{26}$$

where c_2 and c_3 are positive constants. For properchoice $(c_3/\sqrt{c_2})=1$ and$(\sqrt{c_2}/6)=\alpha$ (constant), we have

$$V=(\cosh\alpha t)^6, \tag{27}$$

and the scale factors A, B and C, (from Equation (15)) have the values

$$A=B=C=(\cosh(\alpha t))^2. \tag{28}$$

The line element (2) becomes

$$ds^2=-dt^2+(\cosh(\alpha t))^4(dx^2+dy^2+dz^2). \tag{29}$$

This is Bianchi type I perfect fluid cosmological model in the absence of dark energy in Bimetric theory of gravitation.

It is to be noted that this model has hyperbolic geometry. At $t=0$, $V=A=B=C=1$ and theyarehyperbolicallyincreasinginnaturewithincreasingtime$t$andattaininfinitevalues, as$t\to\infty$.Thisshowsthatthemodelstartswithnonzerovolumeandthevolumeandscale factors are hyperbolically increasing with increase in time t and they go over to infinity at a later stage.

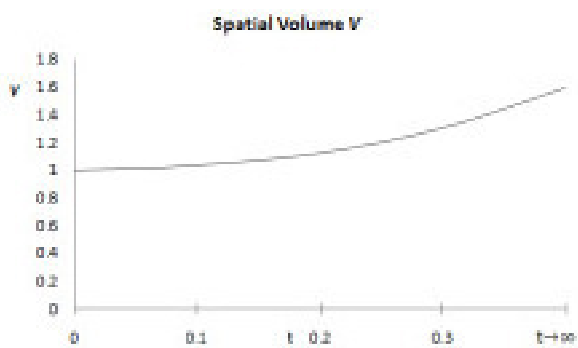

Figure 1. V vs$t$

This hyperbolic geometrical view point of the model will definitely be a benefit to the mathematical and physical community and the people of observational data to search such type of geometry. Recently, Ungar (2009) observed the hyperbolic geometry view point of Einstein's special relativity.

The energy density ρ_{PF} and the isotropic pressure p_{PF} of the perfect fluid model are

$$\rho_{PF}=3p_{PF}\quad c_1/(\cosh(\alpha t))^{4/3}. \tag{30}$$

The physical quantities like scalar expansion θ,anisotropic parameter A, the shear scalar σ and the deceleration parameter q have been calculated as

$$\theta = 6\alpha\tanh(\alpha t), \quad (31)$$

$$A = \sigma = 0, \quad (32)$$

$$q = -\left(1/2(\sinh(\alpha t))^2\right) - 1. \quad (33)$$

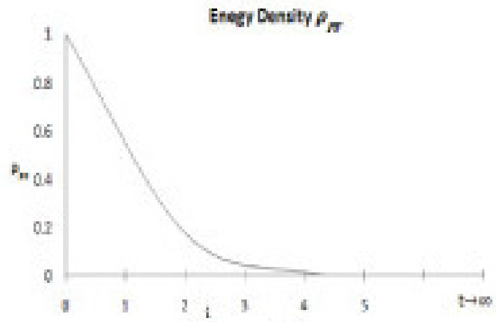

Figure 2. ρ_{PF} vs t

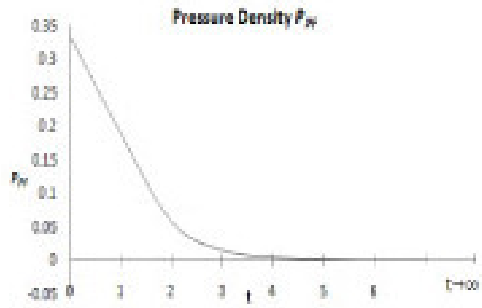

Figure 3. p_{PF} vs t

Figure 2 and Figure 3 show the nature of density and pressure (in similar nature) of the perfect fluid model. At t=0, pressure and density attain the maximum value and theyare decreasing very fast as $t$is increasing, and approaches zero value for 4.5(appro.) $< t < \infty$. Thisshowsthatthisradiatinguniversehasveryhighdensityandpressureinthebeginning and its density and pressure go on decreasing and attain zero value for $t \geq 4.5$ i.e., this radiating universe has the matter for some interval of time $0 \leq t \leq 4.5$ (appro.) and the model admit the vacuum case forever for $t \geq 4.5$.

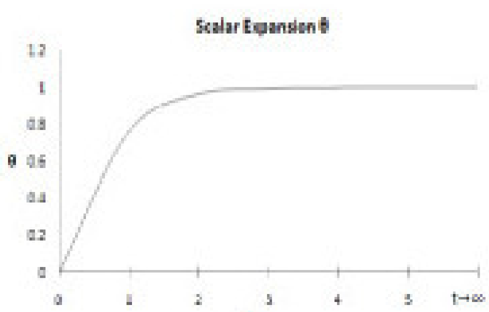

Figure 4. θ vs t

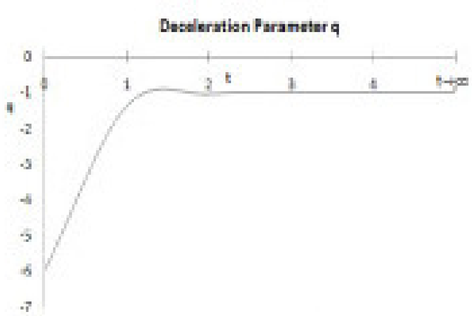

Figure 5. q vs t

The graph of scalar expansion $\theta$is hyperbolic tangential. At t = 0, θ= 0 and it is increasin hyperbolic tangentially and attains the finite value when $t \to \infty$. This shows that the mode

It is well known that the universe underwent an accelerating expansion right after the big-bang (Inflation Era) and at high red shifts (Dark Energy Era). The universe must decelerate *(q>0)* when radiation dominates its dynamics. Figure 5 reflects quite unpleasant nature of deceleration parameter q which has more negative value in the beginning which shows that the model starts with highly accelerating phase and acceleration is slowing down continuously. We are putting this argument on the basis of our mathematical results and this argument may be unpleasant since nobody knows the secret of the nature and the wonder of the physics of the universe.

Case (iii) For $\gamma=1$, Equation (25) has a solution

$$V = \beta \left(\cosh(\sqrt{c_2}\, t) + 1\right), \tag{34}$$

where β is positive constant.
From Equations (15), we write

$$A = B = C = (\beta(\cosh\ t+1))^{1/3}. \tag{35}$$

The required line element (2) is

$$ds^2 = -dt^2 + (\beta(\cosh\sqrt{c_2}\, t+1))^{2/3}(dx^2+dy^2+dz^2). \tag{36}$$

This is the Bianchi type I cosmological model with perfect fluid (stiff matter) in biometric theory of gravitation. This Zeldovich universe has volumetric hyperbolic expansion (as shown in Figure 6 similar to that of the radiating universe case (ii). The scale factors also have similar behavior as case (ii). The model starts with nonzero volume and volume increases in hyperbolic cosine nature and has infinite values at the finalstage.

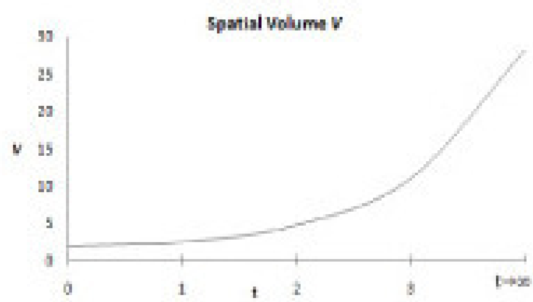

Figure 6. V vs$t$

The energy density ρ_{PF} and the pressure p_{PF} of the perfect fluid (stiff matter) are

$$\rho_{PF} = p_{PF} = c_1 / \beta^2(\cosh(\sqrt{c_2}\, t)+1)^2. \tag{37}$$

The physical parameters are

$$\theta = \frac{6\,\alpha\sinh(\sqrt{c_2}\,t)}{(\cosh(\sqrt{c_2}\,t)+1)}\,, \tag{38}$$

$$A = \sigma = 0, \tag{39}$$

$$q = -\frac{3}{\cosh(\sqrt{c_2}\,t)-1} - 1. \tag{40}$$

In this perfect fluid Zel'dovich universe, it is observed that all the physical parameters $\rho_{pf}, p_{pf}, \theta$ and decelerating parameter q behave in similar nature as that of case (ii) of radiating universe and having the nature of hyperbolic functions as explained earlier in case (ii) and there is no new contribution regarding the geometrical and physical behavior of these parameters, whose graphs are shown below.

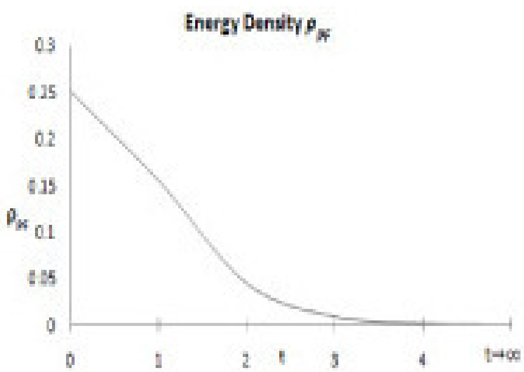

Figure 7. ρ_{pf} vs t

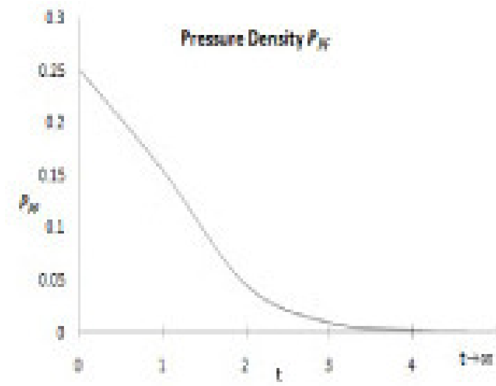

Figure8. p_{pf} vs t

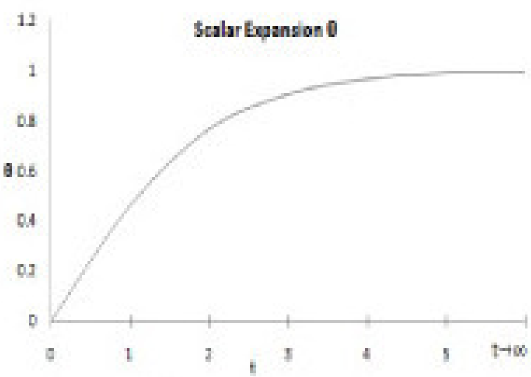

Figure 9. θ vs t

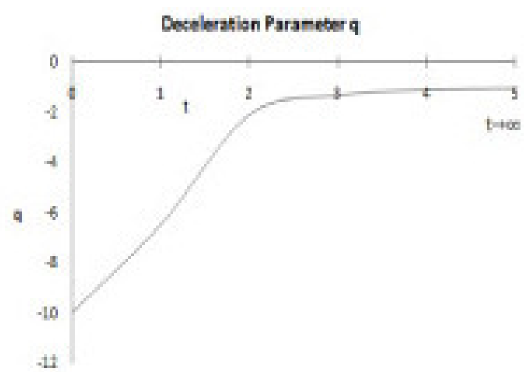

Figure 10. q vs t

It is important to say that in these perfect fluid models (29) and (36) corresponding to radiating universe and Zel'dovich universe, the geometry and all the physical parameters of the models behave hyperbolically in nature, since hyperbolic geometric functions are present in it. Therefore, the geometrical and physical properties of the models with physical parameters have been studied from hyperbolic geometric view point. This is the remarkable point observed in the geometry of the model that the model has hyperbolic geometry and it is helpful to the people of observational data to search such type of universe.

4. Summary

1. Theperfectfluidmodelscorrespondingto $\gamma=1/3$(radiating)and $\gamma=1$(stiffmatter)are hyperbolicgeometricinnature.Theperfectfluidmodelcorrespondenceto $\gamma=0$(dust) does notexist.

2. All the physical matters in these perfect fluid models obey the graph of hyperbolic geometric functions and therefore their natures have been studied from hyperbolic geometric viewpoint.
3. These perfect fluid models have volumetric hyperbolicexpansion.

4. These perfect fluid models are isotropize in nature withoutshear.

5. These perfect fluid models are highly accelerating and acceleration of the model goes on increasing as time t is increasing and has the constant acceleration at later stages of time$t$.

6. Conclusion

Bianchi type-I cosmological models corresponding to perfect fluid in Bimetric theory of gravitation have been deduced. The perfect fluid model has hyperbolic geometry and all its physical parameters are also hyperbolic in nature and therefore they have been studied from a hyperbolic geometric view point.

All these models are isotropize and shear-less. Other geometrical and physical behaviors of the models have been studied.

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