International Journal of Researches In Biosciences, Agriculture & Technology
May 2015
Issue-3, Volume-II
ISSN No. (Online) 2347-517X

EVALUATION OF BIANCHI TYPE VI_0 COSMOLOGICAL MODELS WITH DARK ENERGY IN BIMETRIC THEORY OF GRAVITATION

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Abstract:-
In this paper we discuss study of ***cosmological models with dark energy in bimetric theory of gravitation*** by evaluating Rosen's Field equation with dark is energy is in the form of quintessence and chaplygin gas here the model is completely stands for the dark energy objects. The model shear less and isotopic in nature. The Quintessnece model goes over to vacuum universe. The Chaplygin gas model is isotopicaly same in all direction and it is Shear less.

1 Introduction

There is a lot of attraction to expose the universe by investigating the cosmological models of the universe with dark energy. The observations of the accelerating expansion of the universe strongly suggested that near about 96% of the total energy contain of the universe is exotic in nature out of which 73% is believed as dark energy (gravitationally repulsive in nature), 23% dark matter which is in attractive in nature and remaining 4% of real baryonic matter known by particle theory verified by the observational data of Perlmutter et al. (1998, 1999), Riess et al. (1998), Knop et al. (2003), the observations of Wilkinson Microwave Anisotropic Probe (WMAP) satellite experiment of Riess et al. (2004), Bennett C. L. et al. (2003), Spergel et al. (2003) and Sloan Digital Sky Survey (SDSS) experiment of Abazajian et al. (2003,2004), Tegmark et al. (2004), Adelman-McCarthy et al. (2005) and Seljak et al. (2005) and they have observed that our universe is undergoing in accelerating expansion due to dark energy. The nature of dark energy, can originate from various fields, a canonical scalar field (quintessence) discussed by Ratra et al. (1988), Wetterich (1988), Liddle et al. (1999) and Zlatev et al. (1999), a photon field i.e., a scalar field with negative sign of the kinetic term proposed by Caldwell et al. (2002, 2003), Nojiri et al. (2003), Onemli et al. (2004) and Setare et al. (2008) and the quantum combination of quintessence and photon in a unified models studied by Feng et al. (2005, 2006), Guo et al. (2005), Li et al. (2005), Zhao et al. (2006) and Sadeghi et al. (2008). In addition, there are dark energy models include Chaplygin gas realized by Kamenshchik et al. (2001) and Setare et al. (2007, 2009). The equation of state parameter ω is considered as a constant with values –1, 0, +1/3 and +1 corresponding to vacuum fluid, dust fluid, radiation fluid and stiff dominated universe discussed by Kujat et al. (2002), Bartelmann et al. (2005) and variable $\omega(t)$ of time or red shift is considered by Jimenez (2003) and Das et al. (2005). The quintessence model, $\omega > -1$ (explanation of observations of accelerating universe) involving scalar field and phantom model $\omega < -1$ (expansion of universe increases to infinite degree in finite time) give rise time dependant parameter $\omega(t)$ (see Turner et al. (1997), Caldwell et al. (1998), Liddle et al. (1999) and Steinhardt et al. (1999)). Various forms of parameter $\omega(t)$ have been used for dark energy models by many researchers like Saha et al. (2006), Rahnman et al. (2006, 2009, 2010), Mukhopadhyay et al. (2008), Ray et al. (2010), Akarsu et al. (2010, 2010), Sharif et al. (2010), Yadav et al. (2011) and Pradhan et al. (2011). Also a binary mixture of perfect fluid and dark energy have been considered by Saha (2005), Katore et al. (2006) and Singh et al. (2009), to investigate the cosmological models of the universe and studied the different geometrical and physical aspects of the models.

Rosen's (1973, 1975) bimetric theory of gravitation is one of the alternatives to general relativity and it is free from singularities appearing in the big-bang of cosmological models and it obeys the principles of covariance and equivalence of the general relativity. Therefore the peoples are interested to investigate the cosmological models of the universe in bimetric theory of gravitation to study the geometrical and physical behavior of the model. At every point of the space-time, in bimetric theory of gravitation, there are two metrics: one is Riemannian metric

$$ds^2 = g_{ij}\, dx^i\, dx^j \qquad (1.1)$$

and second is flat metric

$$d\eta^2 = \gamma_{ij}\, dx^i\, dx^j \qquad (1.2)$$

in which g_{ij} are gravitational potentials, described the geometry of curved space-time and γ_{ij} refers to the geometry of the empty universe (no matter but gravitation is there) and described the initial forces.
The Rosen's field equations are

$$N_i^j - \frac{1}{2} N\, \delta_i^j = -T_i^j \qquad (1.3)$$

The Bianchi type VI$_0$ models of the universe give a better explanation of some of the cosmological problems such as primordial helium abundance and they also isotropize in a special sense and therefore, an attempt has been made to study this Bianchi type VI$_0$ cosmological model with a binary mixture of perfect fluid and dark energy (quintessence and Chaplygin gas) in bimetric theory of gravitation. We have investigated three different cosmological models corresponding to perfect fluid, perfect fluid with quintessence and perfect fluid with Chaplygin gas in bimetric theory of gravitation. It is seen that the perfect fluid models, are isotropically same in all directions and shear less. The perfect fluid quintessence model has no solution for $\omega_q = -1$. For $\omega_q = 0$, the quintessence model has physical behavior similar to those of perfect fluid model for $\gamma = 0$, in dust case. Quintessence model with $\omega_q = -1/3$, represents dark energy star and has very high rate of expansion in the beginning and has uniform expansion at later stage. The model is shear less and isotropic in nature. Third perfect fluid Chaplygin gas model has a physical significance that it has volumetric exponential expansion and there is uniform distribution of matter at early stage as well as at later stage and it is isotropic in nature without shear. This third model is exponentially expanding with accelerating expansion which supports the observations of Perlmutter et al. (1998), Knop et al. (2003), Spergel et al. (2003, 2007), Tegmark et al. (2004), Komatsu et al. (2009, 2011) and Hinshaw (2013) [WMAP collaboration] and Ade et al. (2013) [Planck Collaboration] as they observed that the universe is continuously expanding with accelerating expansion.

2 Metric and Field Equations

We consider the Bianchi type-VI$_0$ metric in the form

$$ds^2 = dt^2 - a_1{}^2 dx^2 - a_2{}^2 e^{-2m^2 x} dy^2 - a_3{}^2 e^{2m^2 x} dz^2 \qquad (2.1)$$

where the metric functions a_1, a_2 and a_3 are the functions of t only and m is a constant.
The flat metric corresponding to metric (2.2.1) is

$$d\eta^2 = dt^2 - dx^2 - e^{-2m^2 x} dy^2 - e^{2m^2 x} dz^2 \qquad (2.2)$$

The energy momentum tensor T_i^j of the source with perfect fluid and dark energy is given by

$$T_i^j = (p+\rho) u_i u^j - p\delta_i^j \qquad (2.3)$$

where p and ρ are the pressure and energy density of matter respectively. The quantity θ is the scalar expansion which is given by

$$\theta = v^i{}_{|i} \qquad (2.4)$$

and u^i is the flow velocity vector with magnitude

$$g_{ij} u^i u^j = 1 \qquad (2.5)$$

Co-moving coordinate system is to be assumed and so we write

$$\upsilon^1 = \upsilon^2 = \upsilon^3 = 0, \quad \upsilon^4 = 1. \qquad (2.6)$$

Equation (2.3) of energy momentum tensor yield its components as

$$T_1^1 = T_2^2 = T_3^3 = -p \quad \text{and} \quad T_4^4 = \rho \qquad (2.7)$$

The pressure p and the density ρ are related by an equation of state $p=\gamma\,\rho, \quad 0\le\gamma\le 1$

Aftrer salving Rosen's Field equation, We Arrived at

$$\frac{\ddot{a}_1}{a_1}-\frac{\dot{a}_1^2}{a_1^2}-\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_2^2}{a_2^2}-\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_3^2}{a_3^2}=2p \tag{2.8}$$

$$-\frac{\ddot{a}_1}{a_1}+\frac{\dot{a}_1^2}{a_1^2}+\frac{\ddot{a}_2}{a_2}-\frac{\dot{a}_2^2}{a_2^2}-\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_3^2}{a_3^2}=2p \tag{2.9}$$

$$-\frac{\ddot{a}_1}{a_1}+\frac{\dot{a}_1^2}{a_1^2}-\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_2^2}{a_2^2}+\frac{\ddot{a}_3}{a_3}-\frac{\dot{a}_3^2}{a_3^2}=2p \tag{2.10}$$

$$\frac{\ddot{a}_1}{a_1}-\frac{\dot{a}_1^2}{a_1^2}+\frac{\ddot{a}_2}{a_2}-\frac{\dot{a}_2^2}{a_2^2}+\frac{\ddot{a}_3}{a_3}-\frac{\dot{a}_3^2}{a_3^2}=-2\rho \tag{2.11}$$

Adding equations (2.8) and (2.9), we get

$$\frac{\ddot{a}_1}{a_1}-\frac{\dot{a}_1^2}{a_1^2}=\left(p-\rho\right) \tag{2.12}$$

From the equations we write

$$\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}=0$$

and

$$\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_3}{a_3}=0 \tag{2.13}$$

from which, we have

$$a_1=a_2=a_3 \tag{2.14}$$

The spatial volume $V(t)=a_1a_2a_3$ is given by

$$V={a_1}^3 \tag{2.15}$$

By solving above equations, We get

$$\frac{\ddot{V}}{V}-\frac{\dot{V}^2}{V^2}=3(p-\rho) \tag{2.16}$$

We are going to solve the differential equation (2.16) for the volume V

3 Model with Perfect Fluid and Dark Energy

Quintessence model

Let us consider the case when the dark energy is given by quintessence which obeys the equation of state

$$p_q=\omega_q\rho_q,\ -1\le\omega_q\le 0 \tag{3.1}$$

in which ω_q is equation of state parameter corresponds to quintessence dark energy.

From conservation law of energy momentum tensor for quintessence model, the equation can be written as

$$\dot{\rho}_q + \frac{\dot{V}}{V}\left(\rho_q + p_q\right) = 0$$

Using the condition (3.1), we have

$$\dot{\rho}_q + \frac{\dot{V}}{V}\left(\rho_q + \omega_q\,\rho_q\right) = 0$$

$$\Rightarrow \dot{\rho}_q + \frac{\dot{V}}{V}\rho_q\left(1 + \omega_q\right) = 0$$

$$\Rightarrow \frac{\dot{\rho}_q}{\rho_q} + \frac{\dot{V}}{V}\left(1 + \omega_q\right) = 0$$

On integrating, we get

$$\rho_q = \frac{\alpha_q}{V^{(1+\omega q)}}$$

Substituting this value of $\rho_q = \frac{\alpha_q}{V^{(1+\omega q)}}$, in equation (2.4.1), we have

$$p_q = \frac{\omega_q \alpha_q}{V^{(1+\omega q)}}$$

Thus, the pressure p_q and energy density ρ_q corresponds to quintessence model are given by

$$p_q = \frac{\omega_q \alpha_q}{V^{(1+\omega q)}}\ , \qquad \rho_q = \frac{\alpha_q}{V^{(1+\omega q)}} \tag{3.2}$$

where α_q is the constant of integration.

In the view of above equations, we yield

$$\frac{\ddot{V}}{V} - \frac{\dot{V}^2}{V^2} = 3\left(\frac{\alpha}{V^{(1+\gamma)}}(\gamma - 1) + \frac{\alpha_q}{V^{(1+\omega q)}}(\omega_q - 1)\right)$$

Solving the above equation, we get

$$\int \frac{dV}{\sqrt{K_1 V^{1-\gamma} + K_2 V^{1-\omega q} + \eta}} = t + C \tag{3.3}$$

in which η and C are constants of integration and the constants K_1 and K_2 are given by

$$K_1 = \frac{6(\gamma-1)\alpha}{-(\gamma+1)}, \quad K_2 = \frac{6(\omega_q-1)\alpha_q}{-(\omega_q+1)} \tag{3.4}$$

Selecting the constant of integration C zero and $\gamma = 1$, for high range of perfect fluid matter, the integral equation (2.4.3) gives

$$\int \frac{dV}{\sqrt{\eta + K_2 V^{(1-\omega q)}}} = t \tag{3.5}$$

We are discussing the solution for quintessence with the cases of $\omega_q = -1, 0$ and $-\frac{1}{3}$ with high range of perfect fluid with $\gamma = 1$.

Case (i) For $\omega_q = -1,\ \eta = 0,$ In this case, K_2 not defined and hence integral equation (3.5) has no solution. Therefore quintessence model for $\omega_q = -1$, does not exists.

Case (ii) $\omega_q = 0, \eta = 0$, the integral of (3.5) reduces to

$$\int \frac{dV}{\sqrt{6\alpha_q V}} = t \tag{3.6}$$

which gives

$$V = \left(\frac{3}{2}\alpha_q t^2\right) \tag{3.7}$$

Thus, the values of a_1, a_2, a_3 from equation 2.14 is

$$a_1 = a_2 = a_3 = \left(\frac{3}{2}\alpha_q t^2\right)^{\frac{1}{3}} \tag{3.8}$$

The Hubble parameter H and its directional are

$$H = H_1 = H_2 = H_3 = \frac{2}{3t} \tag{3.9}$$

The expressions of the physical quantities θ, A, σ and q are

$$\theta = \frac{2}{t}, \quad A = 0, \quad \sigma^2 = 0, \quad q = \frac{1}{2}. \tag{310}$$

In this case (v) of $\omega_q = 0$ of quintessence model, the model is dusty universe and it is observed that all the

Case (iii) $\omega_q = -\frac{1}{3}, \eta = 0$, the Integral equation yield

$$\int \frac{dV}{\sqrt{12\alpha_q V^{4/3}}} = t \tag{3.11}$$

which gives

$$V = \left(2\sqrt{\frac{\alpha_q}{3}}t\right)^3 \tag{3.12}$$

From equations with this value of V given ,

values of a_1, a_2, a_3 are

$$a_1 = a_2 = a_3 = 2\sqrt{\frac{\alpha_q}{3}}t \tag{3.13}$$

The Hubble parameter H and its directional are

$$H = H_1 = H_2 = H_3 = \frac{1}{t},. \tag{3.14}$$

We expressed the physical quantities as follows

$$\theta = \frac{3}{t}, \quad A = \sigma^2 = q = 0 \tag{3.15}$$

Chaplygin gas model

Let us now consider the case when the dark energy is represented by Chaplygin gas with equation of state given by

$$p_c = -\frac{\mu}{\rho_c} \tag{3.16}$$

in which μ is constant

From conservation of energy momentum for dark energy, chaplygin gas equation , we write

$$\dot{\rho}_c + \frac{\dot{V}}{V}(\rho_c + p_c) = 0$$

Using equation (3.16), we have

$$\frac{2\dot{\rho}_c\rho_c}{\rho_c{}^2 - \mu} + \frac{2\dot{V}}{V} = 0$$

After integrating, we have

$$\rho_c = \sqrt{\frac{\alpha_c}{V^2} + \mu}$$

where α_c is a constant of integration.

Substituting the above value of ρ_C in equation (3.16), we get

$$p_c = -\frac{\mu}{\sqrt{\frac{\alpha_c}{V^2} + \mu}}$$

Thus, the pressure p_c and density ρ_c in this Chaplygin gas model are

$$p_c = -\frac{\mu}{\sqrt{\frac{\alpha_c}{V^2} + \mu}} \quad \text{and} \quad \rho_c = \sqrt{\frac{\alpha_c}{V^2} + \mu} \qquad (3.18)$$

In view of this pressure and energy density of Chaplygin gas together with the pressure and energy density of perfect fluid with equation V becomes

$$\frac{\ddot{V}}{V} - \frac{\dot{V}^2}{V^2} = 3\left(\frac{\gamma\alpha}{V^{(1+\gamma)}} - \frac{\mu}{\sqrt{\frac{\alpha_c}{V^2} + \mu}} - \frac{\alpha}{V^{(1+\gamma)}} - \sqrt{\frac{\alpha_c}{V^2} + \mu}\right)$$

$$\Rightarrow \frac{\ddot{V}}{V} - \frac{\dot{V}^2}{V^2} = 3\left(\frac{\alpha}{V^{(1+\gamma)}}(\gamma - 1) - \frac{\mu}{\sqrt{\frac{\alpha_c}{V^2} + \mu}} - \sqrt{\frac{\alpha_c}{V^2} + \mu}\right)$$

In the limit of high range of perfect fluid matter density $\gamma = 1,$ the above differential equation becomes

$$\frac{d}{dt}\left(\frac{\dot{V}^2}{V^2}\right) = 6\left\{-\frac{\mu V}{\sqrt{\alpha_c + \mu V^2}} - \frac{\sqrt{\alpha_c + \mu V^2}}{V}\right\}\frac{\dot{V}}{V}$$

In particular, $\alpha_c = 0,$ the above equation reduced to

$$\frac{d}{dt}\left(\frac{\dot{V}^2}{V^2}\right) = -12\sqrt{\mu}\,\frac{\dot{V}}{V}$$

On integrating, we get

$$\int \frac{dV}{V\sqrt{b - 12\sqrt{\mu}\log V}} = t + C$$

where b and C are the constants of integration. This integral equation is difficult to solve and hence for simplicity, we are assuming $b = 12\sqrt{\mu}$. Thus, the above equation becomes

$$\frac{1}{(12\sqrt{\mu})^{\frac{1}{2}}}\int\frac{dV}{V\sqrt{1-\log V}}=t+C$$ Putting $\log V=u$ in the above equation (2.4.22), we have

$$\frac{1}{(12\sqrt{\mu})^{\frac{1}{2}}}\int\frac{du}{\sqrt{1-u}}=t+C$$

gives

$$\frac{1}{(12\sqrt{\mu})^{\frac{1}{2}}}\int(1-u)^{-1/2}du=t+C$$

Using binomial expansion for $0\le u\le 1$, $(1-u)^{-1/2}=\left(1+\frac{1}{2}u+\frac{3}{8}u^2+....\right)$ and integrating term by term, the above integral equation gives

$$\frac{1}{(12\sqrt{\mu})^{\frac{1}{2}}}\left(u+\frac{1}{4}u^2+\frac{1}{8}u^3+.........\right)=t+C$$

We neglecting higher power terms of u, we have

$$\frac{1}{(12\sqrt{\mu})^{\frac{1}{2}}}u=t+C$$

$$\Rightarrow\frac{1}{(12\sqrt{\mu})^{\frac{1}{2}}}\log V=t+C$$

OR

$$V=ae^{\sqrt{b}\,t} \tag{3.19}$$

in which $a=e^{\sqrt{12\sqrt{\mu}}\,C}$ and $\sqrt{b}=\sqrt{12\sqrt{\mu}}$ are constants.

The physical quantities θ,σ,A and q are given by

$$\theta=b,\quad A=0,\quad \sigma^2=0,\quad q=-1 \tag{3.20}$$

The quantities H,H_1,H_2,H_3 and θ all are constant. This shows that the rate of expansion of the universe is constant and it is exponentially expanding with accelerating expansion, since the deceleration parameter q is found to be –1.

It is realized that the anisotropic parameter A and magnitude of shear both are zero. This tell us that the model is isotropically same in all directions and it is shear less.

4 Conclusions

The quintessence model with $\omega_q=-1/3$ represents dark energy star. This model again has volumetric power law expansion and its start with zero volume and zero scale factors and has infinite volume at late time.The quintessence model has a very high rate of expansion in the beginning and it turn over to uniform expansion at later stage. Further, the quintessence model goes over to vacuum universe at late time and it shear less and isotropic in nature. Chaplygin gas perfect fluid model has volumetric exponential law expansion and its start with constant volume and has infinite volume at late time. This chaplygin gas model has uniform distribution of dark matter at early and later stage also. The rate of expansion of this chaplygin gas model) is constant and it is exponentially expanding with accelerating expansion, since the deceleration parameter q is found to be -1. It is shear less and isotropically same in all directions.

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