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STUDY OF BIANCHI TYPE VI_0 COSMOLOGICAL MODEL WITH A BINARY MIXTURE OF PERFECT FLUID IN BIMETRIC THEORY OF GRAVITATION

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Abstract:
The purpose of this research is to know the secrete of the nature by evaluating of bianchi type VI0 cosmological model with a binary mixture of perfect fluid in biometric theory of gravitation have been evaluated by solving Rosen's field equation. The perfect fluid model to γ =0,1/3 and 1 in the equation of state have been studied and it is observed that thease model is volumetric power law exapantion and they have decelerating phase without share.
Keyworsd: Riemannian metric, Cosmology, Dark Energy

1 Introduction

Rosen's (1973, 1975) bimetric theory of gravitation is one of the alternatives to general relativity and it is free from singularities appearing in the big-bang of cosmological models and it obeys the principles of covariance and equivalence of the general relativity. Therefore the peoples are interested to investigate the cosmological models of the universe in bimetric theory of gravitation to study the geometrical and physical behavior of the model. At every point of the space-time, in bimetric theory of gravitation, there are two metrics: one is Riemannian metric

$$ds^2 = g_{ij}\, dx^i\, dx^j \qquad (1.1)$$

and second is flat metric

$$d\eta^2 = \gamma_{ij}\, dx^i\, dx^j \qquad (1.2)$$

in which g_{ij} are gravitational potentials, described the geometry of curved space-time and γ_{ij} refers to the geometry of the empty universe (no matter but gravitation is there) and described the initial forces.
The Rosen's field equations are

$$N_i^j - \frac{1}{2} N\, \delta_i^j = -T_i^j \qquad (1.3)$$

where

$$N_i^j = \frac{1}{2}\, \gamma^{\alpha\beta}\, (g^{sj}\, g_{si|\alpha})_{|\beta}, \gamma \qquad (1.4)$$

is the Rosen's Ricci tensor, $N = g^{ij}\, N_{ij}$ is Rosen's scalar.

It is seen that the perfect fluid models, are isotropically same in all directions and shear less.

2. Metric and Field Equations

We consider the Bianchi type-VI_0 metric in the form

$$ds^2 = dt^2 - a_1{}^2 dx^2 - a_2{}^2 e^{-2m^2 x} dy^2 - a_3{}^2 e^{2m^2 x} dz^2 \qquad (2.1)$$

where the metric functions a_1, a_2 and a_3 are the functions of t only and m is a constant.
The flat metric corresponding to metric (2.1) is

$$d\eta^2 = dt^2 - dx^2 - e^{-2m^2 x} dy^2 - e^{2m^2 x} dz^2 \qquad (2.2)$$

The energy momentum tensor T_i^j of the source with perfect fluid and dark energy is given by

$$T_i^j = (p+\rho)u_i u^j - p\delta_i^j \qquad (2.3)$$

where p and ρ are the pressure and energy density of matter respectively. The quantity θ is the scalar expansion which is given by

$$\theta = v^i_{\ |\,i} \qquad (2.4)$$

and u^i is the flow velocity vector with magnitude

$$g_{ij} u^i u^j = 1 \qquad (2.5)$$

Co-moving coordinate system is to be assumed and so we write

$$\upsilon^1 = \upsilon^2 = \upsilon^3 = 0, \quad \upsilon^4 = 1. \qquad (2.6)$$

Equation (2.2.3) of energy momentum tensor yield its components as

$$T_1^1 = T_2^2 = T_3^3 = -p \text{ and } T_4^4 = \rho \qquad (2.7)$$

The pressure p and the density ρ are related by an equation of state $p = \gamma\rho, \quad 0 \le \gamma \le 1$

The Rosen's field equations are

$$\frac{\ddot{a}_1}{a_1} - \frac{\dot{a}_1^2}{a_1^2} - \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_2^2}{a_2^2} - \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_3^2}{a_3^2} = 2p \qquad (2.8)$$

$$-\frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1^2}{a_1^2} + \frac{\ddot{a}_2}{a_2} - \frac{\dot{a}_2^2}{a_2^2} - \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_3^2}{a_3^2} = 2p \qquad (2.9)$$

$$-\frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1^2}{a_1^2} - \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_2^2}{a_2^2} + \frac{\ddot{a}_3}{a_3} - \frac{\dot{a}_3^2}{a_3^2} = 2p \qquad (2.10)$$

$$\frac{\ddot{a}_1}{a_1} - \frac{\dot{a}_1^2}{a_1^2} + \frac{\ddot{a}_2}{a_2} - \frac{\dot{a}_2^2}{a_2^2} + \frac{\ddot{a}_3}{a_3} - \frac{\dot{a}_3^2}{a_3^2} = -2\rho \qquad (2.11)$$

From the above equations , we have

$$a_1 = a_2 = a_3 \qquad (2.12)$$

The spatial volume $V(t) = a_1 a_2 a_3$ is given by

$$V = a_1^{\,3} \qquad (2.13)$$

On differentiating equation (2.23) and using equations (2.12),

We get

$$\frac{\ddot{V}}{V} - \frac{\dot{V}^2}{V^2} = 3(p-\rho) \qquad (2.14)$$

We are going to solve the differential equation (2.14) for the volume V

3 Model with Perfect Fluid

The conservation law for the energy momentum tenser , yield

$$\dot{\rho} = -\frac{\dot{V}}{V}(\rho + p) \tag{3.1}$$

The perfect fluid obeys the equation of state

$$p_{PF} = \gamma \ \rho_{PF}, \quad 0 \le \gamma \le 1, \tag{3.2}$$

Here $\gamma = 0$ (Dust Universe), $\gamma = \frac{1}{3}$ (Radiation Universe), $\gamma = \left(\frac{1}{3}, 1\right)$ (Hard Universe) and $\gamma = 1$ (Zel'dovich Universe or stiff matter).

In a commoving coordinate, the conservation law of energy momentum tensor (2.3.1), for the perfect fluid and dark energy is

$$\dot{\rho}_{DE} + \dot{\rho}_{PF} = -\frac{\dot{V}}{V}\left(\rho_{DE} + \rho_{PF} + p_{DE} + p_{PF}\right) \tag{3.3}$$

from which, we have

$$\dot{\rho}_{DE} + \frac{\dot{V}}{V}\left(\rho_{DE} + p_{DE}\right) = 0 \tag{3.4}$$

and

$$\dot{\rho}_{PF} + \frac{\dot{V}}{V}\left(\rho_{PF} + p_{PF}\right) = 0 \tag{3.5}$$

corresponding to dark energy (DE) and perfect fluid (PF) respectively.

With the help of the equation $p_{PF} = \gamma \ \rho_{PF}, \quad 0 \le \gamma \le 1$ from (3.2) , we write the above equation (3.5) as

$$\dot{\rho}_{PF} + \frac{\dot{V}}{V}\left(\rho_{PF} + \gamma\rho_{PF}\right) = 0$$

$$\Rightarrow \dot{\rho}_{PF} + \frac{\dot{V}}{V}\rho_{PF}(1+\gamma) = 0$$

$$\Rightarrow \frac{\dot{\rho}_{PF}}{\rho_{PF}} + \frac{\dot{V}}{V}(1+\gamma) = 0$$

After integrating, we have

$$\log \rho_{PF} + (1+\gamma)\log V = \log \alpha$$

which leads

$$\rho_{PF} = \frac{\alpha}{V^{(1+\gamma)}}$$

Using $\rho_{PF} = \frac{\alpha}{V^{(1+\gamma)}}$ in equation (3.2), we have the pressure corresponding to perfect fluid as

$$p_{PF} = \frac{\gamma\,\alpha}{V^{(1+\gamma)}}$$

Thus the pressure and the energy density of perfect fluid are given as

$$p_{PF} = \frac{\gamma\,\alpha}{V^{(1+\gamma)}}, \qquad \rho_{PF} = \frac{\alpha}{V^{(1+\gamma)}} \tag{3.6}$$

where α is an integrating constant.

Using these values of p_{PF} and ρ_{PF} from above equation (3.6), the equation (2.14) infer

$$\dot{V} = \pm\sqrt{\beta - \frac{6\alpha(\gamma-1)}{(\gamma+1)} V^{(1-\gamma)}} \tag{3.7}$$

where β is a constant of integration.

This equation (2.3.7) has a solution

$$\int \frac{dV}{\sqrt{\beta - \frac{6\alpha(\gamma - 1)}{(\gamma + 1)} V^{(1-\gamma)}}} = t + C$$

where C is a constant of integration which is taken to be zero. So that

$$\int \frac{dV}{\sqrt{\beta - \frac{6\alpha(\gamma - 1)}{(\gamma + 1)} V^{(1-\gamma)}}} = t \tag{3.8}$$

This integral equation (3.8) is difficult to solve and hence we are assuming particular cases to have the solution.

Particular Cases

Case (i) For $\gamma = 0, \beta = 0$

From equation (3.8), for $\gamma = 0, \beta = 0$, we obtain

$$\int \frac{dV}{\sqrt{6\alpha \mathrm{V}}} = t \tag{3.9}$$

which gives

$$V = \frac{3}{2}\alpha t^2 \tag{3.10}$$

Putting this value of V in the equations (2.14) and (2.15), we get

$$a_1 = a_2 = a_3 = \left(\frac{3}{2}\alpha t^2\right)^{\frac{1}{3}}. \tag{3.11}$$

This equation (3.11) gives the solution of Bianchi Type VI_0 cosmological model with perfect fluid in which there is no role of dark energy. This solution is in power law expansion. The model has the volumetric power law expansion. At early stage of the universe, the volume V of the model is zero and then it is increasing with increasing in time and it tends to infinity at late time. Also the scale factors are zero at early stage and they are increasing with increasing in time t and reaches to infinity at later stage. Thus the perfect fluid model starts with zero volume and zero scale factors and they reaches infinity at final stage in the absence of the dark energy

The Hubble parameter H is defined as the ratio of the rate of change of an average scale factor a with an average scale factor a i.e., $\frac{\dot{a}}{a}$ and it represents the rate of expansion of the universe. In terms of its directionals H_1, H_2 and H_3 along x, y and z axis it defined as

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right)$$

where H_1, H_2 and H_3 are the directionals of H and they are

$$H_1 = \frac{\dot{a}_1}{a_1},\ H_2 = \frac{\dot{a}_2}{a_2} \text{ and } H_3 = \frac{\dot{a}_3}{a_3}.$$

Now using the values of a_1, a_2 and a_3 , from equation (3.11), we write

$$H = \frac{2}{3t}$$

and

$$H_1 = H_2 = H_3 = \frac{2}{3t} \tag{3.12}$$

From equations (2.3.6) and (2.3.10), we write the isotropic pressure p and the energy density ρ of the perfect fluid as

$$p_{\text{PF}} = 0 \quad \text{and} \quad \rho_{PF} = \frac{2}{3t^2} \tag{3.13}$$

We are calculating the physical quantities like expansion scalar θ, anisotropic parameter A, the shear scalar σ and the declaration parameter q for the model with perfect fluid and discussing their geometrical and physical significance in this article.

Substituting the value of H, from equation (3.12) in $\theta = 3H$, the expansion scalar θ have been calculated as

$$\theta = \frac{2}{t}$$

The anisotropic parameter A is defined as

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2$$

OR

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2$$

and using equation (2.3.12), it is

$$A = 0$$

The shear scalar σ is given as

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) = \frac{3}{2}AH^2$$

Using the value of anisotropic parameter $A = 0$, we write

$$\sigma = 0$$

The deceleration parameter q have the value

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{1}{2}$$

Thus, we have

$$\theta = \frac{2}{t} \tag{3.14}$$

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = 0 \qquad (3.15)$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^{\ 2} - 3H^2\right) = \frac{3}{2}AH^2 = 0 \qquad 3.16)$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{1}{2} \qquad (3.17)$$

In this case (i), for $\gamma = 0,\ \beta = 0$ the model represents dusty universe in which the Hubble parameter H and its directionals, the scalar expansion θ all are infinite at early stage and they admits zero values at late time. Thus the rate of expansion is diverges to infinity at early stage and there is no expansion of the model at late epoch of time .The model goes over to vacuum at final stage. The model has no shear and it is isotropize in all directions. The model has decelerating expansion, since the deceleration parameter q is admit positive value, in this dusty universe.

Case (ii) For $\gamma = \frac{1}{3}, \beta = 0,$ the expression (3.8) gives

$$\int \frac{dV}{\sqrt{3\alpha\ V^{\frac{2}{3}}}} = t \qquad (3.18)$$

which yield

$$V = \left(2\sqrt{\frac{\alpha}{3}}\,t\right)^{\frac{3}{2}} \qquad (3.19)$$

Thus, the value of a_1 is calculated from equation (2.14) as

$$a_1 = V^{1/3} = \left(2\sqrt{\frac{\alpha}{3}}\,t\right)^{\frac{1}{2}}$$

Using relation (2.13), we have

$$a_1 = a_2 = a_3 = \left(2\sqrt{\frac{\alpha}{3}}\,t\right)^{\frac{1}{2}} \qquad (3.20)$$

The solution (3.20) represents radiating universe which has volumetric power expansion, in which the nature of the volume V and the scale factors are same as that of case (i) of dusty universe.

The Hubble parameter H and its directionals are

$$H = H_1 = H_2 = H_3 = \frac{1}{2t} \qquad (3.21)$$

From equations (3.6) and (3.19), the isotropic pressure p_{PF}, and the energy density ρ_{PF}, corresponding to perfect fluid are given by

$$p_{PF} = \frac{1}{4t^2} \quad \text{and} \quad \rho_{PF} = \frac{3}{4t^2} \qquad (3.22)$$

We calculating the physical quantities θ, A, σ and q and discussing their geometrical and physical interpretation.

Substituting the value of H, from equation (3.21) in $\theta = 3H$, the expansion scalar θ is

$$\theta = \frac{3}{2t}$$

The anisotropic parameter A is defined as

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2$$

Using equation (3.21), we write

$$A = 0$$

The shear scalar σ is given as

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^{\,2} - 3H^2\right)$$

Using the value of anisotropic parameter $A = 0$, we write

$$\sigma = 0$$

The deceleration parameter q have the value

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = 1$$

Thus, we have

$$\theta = \frac{3}{2t}, \quad A = 0, \quad \sigma^2 = 0, \quad q = 1 \qquad (3.23)$$

In this radiating universe, all the physical parameters $H, H_1, H_2, H_3, \rho, \theta, A, \sigma$ and q having the similar nature as that of case (i) of dusty universe, except the nature of isotropic pressure p which is non zero. At late time, this radiating universe switched over to vacuum universe.

Case (iii) For $\gamma = 1$ and $\beta > 0$ (for $\beta \leq 0$, the integral equation (3.8) is meaningless), from equation (2.3.8), we have

$$\int \frac{dV}{\sqrt{\beta}} = t \tag{3.24}$$

On integrating above equation (2.3.24), we have

$$V = \sqrt{\beta}\ t \tag{3.25}$$

Using the value of V from this equation (3.25), the value of a_1 is calculated from equation (2.14) as

$$a_1 = V^{1/3} = \left(\sqrt{\beta}\ t\right)^{\frac{1}{2}}$$

and then the relation (2.13) gives

$$a_1 = a_2 = a_3 = (\sqrt{\beta}\ \mathrm{t})^{\frac{1}{3}}$$

Thus the values of a_1, a_2, a_3 are

$$a_1 = a_2 = a_3 = (\sqrt{\beta}\ \mathrm{t})^{\frac{1}{3}} \tag{3.26}$$

This solution (3.26) represents Zel'dovich universe whose volume and the scale factors behave similar to that volume and scale factors of case (i) of dusty universe and case (ii) of radiating universe.

From equations (.3.6) and (3.25), the isotropic pressure p_{PF} and energy density ρ_{PF}, in this case are

$$p_{PF} = \rho_{PF} = \frac{\alpha}{\beta t^2} \tag{3.27}$$

Using the relation $H_1 = \frac{\dot{a}_1}{a_1}, H_2 = \frac{\dot{a}_2}{a_2}, H_3 = \frac{\dot{a}_3}{a_3}$ and equation (3.26), we write

$$H_1 = H_2 = H_3 = \frac{1}{3t}$$

The Hubble parameter H is given by

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \frac{1}{3t}$$

Thus, the Hubble parameter H and its directionals $H_1,\ H_2$ and H_3 are

$$H = H_1 = H_2 = H_3 = \frac{1}{3t} \qquad (3.28)$$

We calculating the physical quantities θ, A, σ and q , for $\gamma = 1$ and for $\beta > 0$ and discussing their geometrical and physical interpretation as follows.

Substituting the value of H , from equation (3.28) in $\theta = 3H$, the expansion scalar θ is

$$\theta = \frac{1}{t}$$

The anisotropic parameter A is defined as

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2$$

and using equation (2.3.28), we have

$$A = 0$$

The shear scalar σ is given by

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^{\ 2} - 3H^2\right)$$

and using the value of anisotropic parameter $A = 0$, we write

$$\sigma = 0$$

The deceleration parameter q takes the form

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = 2$$

Thus, the physical quantities θ, A, σ and q have the values

$$\theta = \frac{1}{t}, \quad A = 0, \quad \sigma^2 = 0, \quad q = 2. \qquad (3.29)$$

In this Zel'dovich universe, we realized that the nature of all physical parameters $p, \rho, H, H_1, H_2, H_3, \theta, A, \sigma$ and q is similar to those of the nature of physical parameters of case (ii) of radiating universe and they are not gaining any new ideas.

In all these three models dusty universe, radiating universe and Zel'dovich universe, the models have volumetric power law expansion. It is seen that, the mean Hubble parameter H and its directionals are the functions of t and inversely proportional to time t. At $t = 0$, the quantities H_1, H_2, H_3 and H tends to infinity and they approaches to zero, when $t \to \infty$. From this, we say that the mean Hubble parameter H and its directionals diverges to infinity, at early stage of universe and they approaches to zero at late epoch of time. The scalar expansion θ having infinite value at $t = 0$ and it approaches to zero when $t \to \infty$. This infer that in the beginning of the universe, there is infinite scalar expansion and expansion of the model decreases when time t increases and there is no expansion at late time. Further, the anisotropic parameter A has the zero value. This suggested that the model is isotropic in all directions. The magnitude of shear σ is zero infer shear less model. It is noticed that in all three cases, the declaration parameter q in the model of perfect fluid is always positive. From this, we conclude that the model has the decelerating expansion always in all cases. Thus the universe has volumetric power law expansion and decelerating phase, in all these three cases of dusty, radiating and Zel'dovich in perfect fluid.

3 Conclusions

Three different cases of perfect fluid have been studied. It is seen that in all the cases of perfect fluid model, the model has volumetric power law expansion and decelerating phase. Perfect fluid model is isotropically same in all directions and it is shear less in all three cases.

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