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# MAGNETIZED STRING COSMOLOGICAL MODEL IN ROSEN'S BIMETRIC THEORY OF GRAVITATION

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**ABSTRACT***:*
We have investigated Locally Rotationally Symmetric Bianchi Type-II Magnetized String Cosmological Model by solving the field equations of Bimetric Theory of Gravitation. The model has volumetric hyperbolic expansion and expansion is always in accelerating phase. Other geometrical and physical behavior of the model have been studied.
***KEY WORDS****:* Gravitation, Magnetic field, String theory, Cosmology and Bianchi type Models.

## INTRODUCTION:

The occurrence of magnetic field on galactic scale is a well established fact today and anisotropic magnetic field models have significant contribution in the evolution of galaxies and stellar objects. Harrison [1] has described that magnetic field could have a cosmological origin. Melvin [2] has claim that during the evolution of Universe, the matter is in highly ionized state and due to smooth coupling with field it form neutral matter as a result of universe expansion. Strong magnetic field can be created due to adiabatic compression in clusters of galaxies. Large-scale magnetic field gives rise to anisotropies in the universe. Therefore, the presence of magnetic field in anisotropic string universe is not unrealistic. Asseo and Sol [3] emphasized the importance of the Bianchi Type II universe. Roy and Banerjee[4], Yadav et al.[5] and Bali[6] have investigated the LRS Bianchi type II cosmological models representing the clouds as well as massive strings.

The field equations of Rosen's [7, 8] bimetric theory of gravitation are

$$N_i^j - \frac{1}{2} N\delta_i^j = -8\pi k T_i^j \qquad (1)$$

where $N_i^j = \frac{1}{2}\gamma^{pr}\left(g^{sj}g_{si}|_p\right)|_r$ , $N = N_i^i$ , $k = \sqrt{\frac{g}{\gamma}}$ together with $g = det(g_{ij})$ and $\gamma = det(\gamma_{ij})$, Here the vertical bar $(\,|\,)$ stands for γ-covariant differentiation and $T_i^j$ is the energy–momentum tensor of matter field. The Bimetric theory of gravitation is free from the singularities that occur in general relativity that was appearing in the big–bang in cosmological models. Several aspects of bimetric theory of gravitation have been studied and investigated many Bianchi type cosmological models in it by many researchers [9-21]. Locally Rotationally Symmetric Bianchi Type-II Magnetized String Cosmological Model with Bulk Viscous Fluid have been studied by Atul Tyagi et al.[22] in Einstein general relativity and we plan to study this model in Rosen's bimetric theory of gravitation on the ground of its geometrical and physical behavior.

It is seen that this Locally Rotationally Symmetric Bianchi Type-II Magnetized Magnetized Cosmological Model in Rosen's Bimetric Theory of Gravitation has volumetric hyperbolic expansion and geometrical and physical behavior of all physical parameters are hyperbolic in nature which is not in general relativity. This hyperbolic geometric view point of our model will definitely help to the people of physicist community to search such type of universe.

## THE METRIC AND FIELD EQUATIONS

We consider the LRS Bianchi Type-II metric in the form

$$ds^2 = -dt^2 + A^2(dx^2 + dz^2) + B^2(dy - xdz)^2, \quad (2)$$

where $A$ and $B$ are functions of cosmic time t-alone.

The flat metric corresponding to metric (2) is

$$d\eta^2 = -dt^2 + (dx^2 + dz^2) + (dy - dz)^2. \quad (3)$$

The energy momentum tensor $(T_i^j)$ for a cloud of strings with bulk viscous fluid and electromagnetic field $(E_i^j)$ is given by

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \xi v^l{}_{;l}\left(v_i v^j + g_i^j\right) + E_i^j, \quad (4)$$

where $\rho$ is the rest energy density of the cloud strings, λ is the string tension density and $\xi$ is the coefficient bulk viscosity, $\theta$ is the expansion scalar, $v^i = (0,0,0,1)$ is the space-like four velocity vector and the direction of the string $x^i$ is choosing $x^i = (\frac{1}{A}, 0, 0, 0)$, the time-like vector such that

$$v_i v^j = -x_i x^j = -1 \quad (5)$$

$$v^i x_i = 0, \quad (6)$$

and electromagnetic field $(E_i^j)$ is

$$E_i^j = \bar{\mu}\left[|h|^2\left(v_i v^j + \frac{1}{2} g_i^j\right) - h_i h^j\right]. \quad (7)$$

The magnetic flux vector is given by

$$h_i = \frac{\sqrt{-g}}{2b} \epsilon_{ijkl} F^{kl} v^j, \quad (8)$$

where $\bar{\mu}$ is the magnetic permeability, $F^{kl}$ is the electromagnetic field tensor, $\epsilon_{ijkl}$ is the Levi-civita tensor, magnetic field is along the x-direction, so that $F_{23}$ is the only non-vanishing component of $F^{sp}$ and $h_1 \neq 0, h_2 = h_3 = h_4 = 0$. Due to assumption of infinite electrical conductivity, we have $F_{14} = F_{24} = F_{34} = 0$ and $F_{23} \neq 0$.

The particle is loaded on the string and its density $\rho_p$ is defined by

$$\rho_p = \rho - \lambda. \quad (9)$$

From Maxwell's equation, $F_{[ij,k]} = 0$, we write

$$F_{23} = -F_{32} = H = \text{Constant}. \quad (10)$$

Equation (8) leads to

$$h_1 = \frac{H}{\bar{\mu}\, B} \quad (11)$$

Using equation (11), equation (7) yield

$$E_1^1 = -\frac{H^2}{2\,\bar{\mu} B^2 A^2} = -E_2^2 = -E_3^3 = E_4^4 \quad (12)$$

Rosen's field equations (1), for the metric (2) and (3) becomes

$$\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} = 16\pi A^2 B\left(\lambda + \xi\theta + \frac{H^2}{2\,\bar{\mu} B^2 A^2}\right), \quad (13)$$

$$2\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - 2\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} - \frac{B^2}{A^2} = 16\pi A^2 B\left(\xi\theta - \frac{H^2}{2\,\bar{\mu} B^2 A^2}\right), \quad (14)$$

$$\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} + \frac{B^2}{A^2} = 16\pi A^2 B\left(\xi\theta - \frac{H^2}{2\,\bar{\mu} B^2 A^2}\right), \quad (15)$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - 2\frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2} = 16\pi A^2 B\left(\rho + \frac{H^2}{2\,\bar{\mu} B^2 A^2}\right), \quad (16)$$

The law of conservation of energy momentum tensor

$$T^{ij}\big|_j = 0$$

yield

$$\dot{\rho} = \lambda\left(2\frac{\dot{A}}{A}\right) - (\rho - \xi\theta)\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \quad (17)$$

where $\dot{A} = \frac{\partial A}{\partial t}$, $\dot{B} = \frac{\partial B}{\partial t}$, $\ddot{A} = \frac{\partial^2 A}{\partial t^2}$, $\ddot{B} = \frac{\partial^2 B}{\partial t^2}$.

## SOLUTION OF FIELD EQUATIONS WITH PHYSICAL QUANTITIES

These equations (13-17) are five differential equations in five unknowns $A, B, \rho, \lambda$ and $\xi$, (as $\theta$ is in terms of $A$ and $B$). Therefore, this system of five differential equations (13-17) determine unique solution.

After solving equations (14) and (15) we get,

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}\right) - \frac{d}{dt}\left(\frac{\dot{B}}{B}\right) = \frac{B^2}{A^2}, \quad (18)$$

and from equations (13) - (16), we write,

$$-2\frac{d}{dt}\left(\frac{\dot{A}}{A}\right) = 16\pi A^2 B(\lambda + \xi\theta - \rho) \quad (19)$$

With this equation (19), the differential equations (13), (15) and (16) yield

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$$\frac{d}{dt}\left(\frac{\dot{A}}{A}\right) = \frac{B^2}{A^2} \qquad (20)$$

From equations (18) and (20), we get

$$\frac{d}{dt}\left(\frac{\dot{B}}{B}\right) = 0 \qquad (21)$$

which yield

$$B = e^{(lt+m)} \qquad (22)$$

where $l\ (>0)$ and $m$ are constants of integration.

Using this value (equation (22)) of $B$, from equation (20), after straightforward calculations, we write the value of $A$ as

$$A = \frac{1}{2}\left[e^{(lt+2m-t-c_2)} + e^{(lt+t+c_2)}\right] \qquad (23)$$

where $c_1$, $c_2$ and $m$ are constants.

For simplicity we assume $c_2 = m$. So that

$$A = e^{(lt+m)} \cosh t \qquad (24)$$

Hence metric (2) reduces to

$$ds^2 = -dt^2 + e^{2(lt+m)}[\cosh^2 t\,(dx^2 + dz^2) + (dy - xdz)^2]. \qquad (25)$$

This is the required metric represents locally rotationally symmetric Bianchi type-II magnetized string cosmological model with bulk viscous fluid in bimetric theory of gravitation and it which is free from singularity. It is to be noted that the behavior of our model reflects by hyperbolic geometric functions and therefore the geometry of our model is hyperbolic in nature. This is the interesting point in the geometry of the model that our model has hyperbolic geometry, and it is helpful to the people of physicist community to search such type of geometry.

The spatial volume $V$, energy density $\rho$, the string tension density $\lambda$, the particle density $\rho_p$, the bulk viscosity $\xi$, the scalar of expansion $\theta$ and the shear tensor $\sigma$ for the model (25) are

$$V = e^{3(lt+m)}(\cosh t)^2\,, \qquad (26)$$

$$\rho = \left(\frac{1-\tanh^2 t}{e^{3(lt+m)}}\right)\left[\frac{(1-\tanh^2 t)}{8\pi} - \frac{H^2}{2\bar{\mu}e^{(lt+m)}}\right], \qquad (27)$$

$$\lambda = \left(\frac{\tanh^2 t-1}{e^{3(lt+m)}}\right)\left[\frac{(1-\tanh^2 t)}{16\pi} + \frac{H^2}{\bar{\mu}e^{(lt+m)}}\right], \qquad (28)$$

$$\theta = v^{l}\big|_{l} = 2\tanh t + 3l, \qquad (29)$$

$$\xi = \frac{(1-\tanh^2 t)}{(2\tanh t+3l)e^{3(lt+m)}}\left[\frac{(1-\tanh^2 t)}{16\pi} + \frac{H^2}{2\bar{\mu}e^{(lt+m)}}\right], \qquad (30)$$

$$\rho_p = \left(\frac{1-\tanh^2 t}{e^{3(lt+m)}}\right)\left[\frac{3(1-\tanh^2 t)}{16\pi} + \frac{H^2}{2\bar{\mu}e^{(lt+m)}}\right], \qquad (31)$$

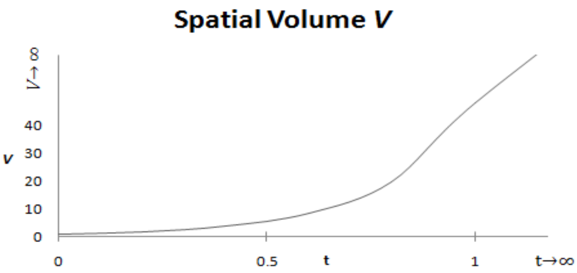
$$\sigma = \frac{1}{\sqrt{3}}(2\tanh t + 3l)^{\frac{1}{2}} \qquad (32)$$

The deceleration parameter $(q)$ for the model (25) is given by

$$q = -\left[1 + \frac{{}^2\!/_3(1-\tanh^2 t)}{\left({}^2\!/_3\tanh t + l\right)^2}\right] \qquad (33)$$

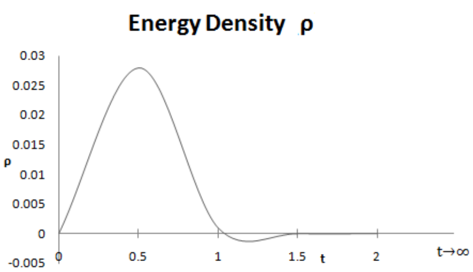
We are going to study the geometrical and physical behavior of all these physical parameters by assuming $l = 1$ and $m = 0$ in next section.

## THE GEOMETRICAL AND PHYSICAL SIGNIFICANCE OF THE MODEL



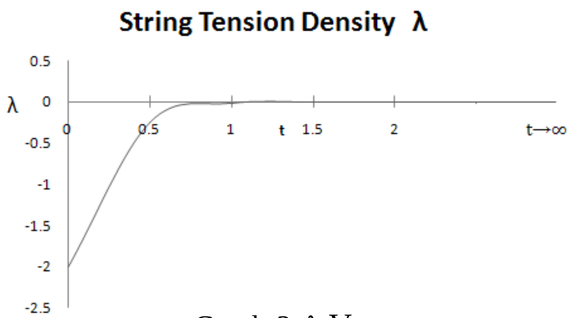

Graph-1: $V$ Vs $t$

The model has volumetric hyperbolic expansion. At $t = 0$, the volume $V$ attain the value one and it is hyperbolically increasing function of time $t$, and admit the infinite value, when $t \to \infty$ (shown in Graph – 1). This shows that the model start with constant (nonzero) volume and volume of the model increasing hyperbolically with increasing time $t$ and tends to infinite at later stage of time $t$.



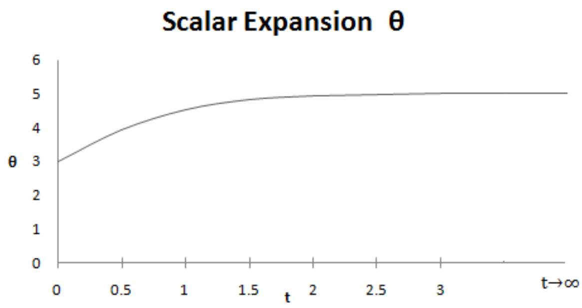

Graph-2: ρ Vs $t$

The behavior of the geometry and all the physical parameters of the model are governed by hyperbolic geometric functions of time $t$. The density parameter behave hyperbolic in nature (Graph–2). At early stage $t = 0$, the density is zero and increasing rapidly as $t$ increases (slowly), reaches the peak point and suddenly coming down and attain the zero value at $t = 1.06$. In the meanwhile for small interval of time, it become negative and then attain the zero value forever for whole domain of time $t$ (Graph–2). This shows that the model with bulk viscous fluid start with zero density and attain the maximum density of the matter at time $t = 0.5$ and then density decreasing and it vanish at $t = 1.06$ . For small interval of time, model suddenly does not exist and then model admit vacuum case from $t \approx 1.5$ onwards.



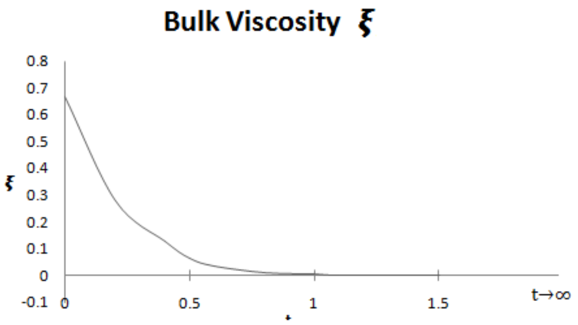

Graph-3: λ Vs $t$

String tension density λ in the model is negative $0 \leq t < 1$ and it is zero for $1 \leq t < \infty$. This shows that the string tension density in the model do not exists in the domain of time $0 \leq t < 1$ and for the whole domain $1 \leq t < \infty$ , string tension density attain zero value (see graph-3)



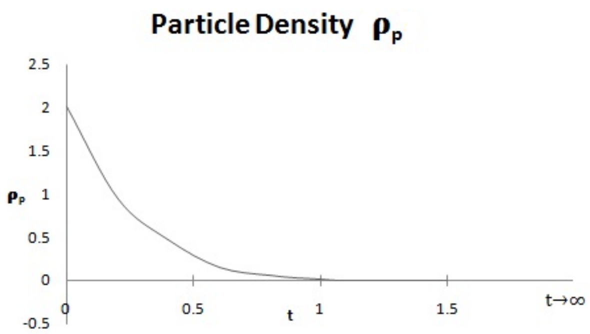

Graph-4: θ Vs $t$

The graph of scalar expansion θ is hyperbolic tangential in nature. At $t = 0$, the scalar expansion attain the constant value three and it is hyperbolic tangentially increasing with increase in time $t$ and goes to finite value, when The graph of scalar expansion θ is hyperbolic tangential in nature. At $t = 0$, the scalar expansion attain the constant value three and it is hyperbolic tangentially increasing with increase in time $t$ and goes to finite value, when $t \rightarrow \infty$. This shows that in the beginning, model has expansion and it is expanding hyperbolic tangentially and attain the finite value at later stage of time $t$ (Graph–4).



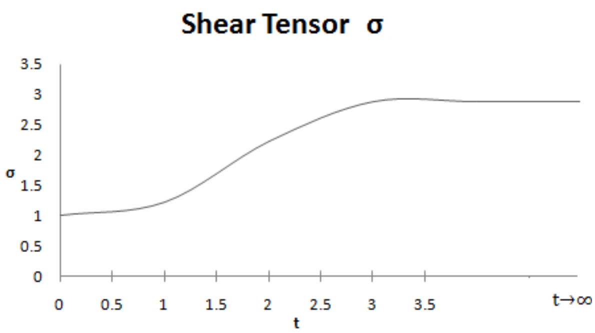

Graph-5: $\xi$ Vs $t$

Graph-5 shown the behavior of bulk viscous fluid with respect to time $t$. At $t = 0$, the coefficient of bulk viscosity ξ retain the highest value and it is gradually decreasing as the time $t$ increasing for $0 \leq t \leq 1$ and attain the zero value for $t \geq 1$. This shows that in the beginning, the model has highest bulk viscosity of the fluid and gradually it is slowing down and there is no bulk viscous fluid in the model at final stage.



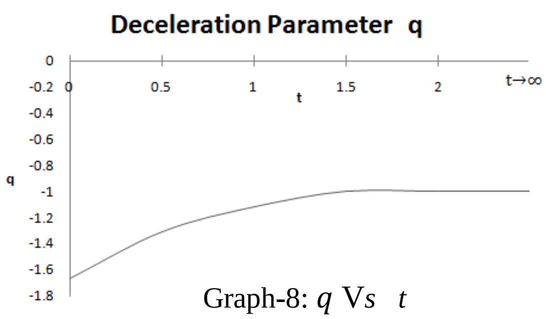

Graph-6: $\rho_p$ Vs $t$

The particle density $\rho_p$ in the model is shown in the Graph-6, and it has similar behavior ashat of coefficient of bulk viscosity.




Graph-7: $\sigma$ Vs $t$

The model always has shear. At $t = 0$, the shear $\sigma$ retain the value one and it is increasing in the sense of graph of hyperbolic tangential function as time $t$ increasing and has finite value when $t \to \infty$. This shows that the model starts with constant shear and it is hyperbolic tangentially increasing with increases in time $t$ and goes over to finite value at final stage of time $t$.




Graph-8: $q$ Vs $t$

The deceleration parameter $q$ in the model is always appeared with negative values. At $t = 0$, the deceleration parameter $q$ attain the value $q = -1.667$ and it is gradually increasing as $t$ increasing and reaches the value $q = -1$ at $t = 1.5$. For $t \geq 1.5$ , $q = -1$ forever. This shows that the model has accelerating hyperbolic expansion for $0 \leq t < 1.5$ and it has constant accelerating expansion for $t \geq 1.5$. The model has no decelerating phase of expansion.

Further the ratio $\frac{\sigma}{\theta} \neq 0$ as $t \to \infty$ shows that model is not isotropize.

**CONCLUSION**

We have investigated Locally Rotationally Symmetric Bianchi Type-II Magnetized String Cosmological Model with Bulk Viscous Fluid in Bimetric Theory of Gravitation and studied its geometrical and physical behavior. The model has volumetric hyperbolic expansion and expansion is always in accelerating phase. All the physical parameters and geometry behave hyperbolic in nature, as the hyperbolic geometric function appears in it. The people of physicist community will definitely attract towards this point to search such type of geometry of the universe.

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