INTERNATIONAL JOURNAL OF RESEARCHES IN BIOSCIENCES, AGRICULTURE AND TECHNOLOGY
© VISHWASHANTI MULTIPURPOSE SOCIETY (Global Peace Multipurpose Society) R. No. MH-659/13(N)
www.vmsindia.org

Bianchi Type VI$_o$ Magnetized Cosmological Model in $f(R,T)$ Theory of Gravitation

V. G. Mete[1] and K. R. Mule[2]
[1]Department of Mathematics, R.D.I.K. & K.D. College, Badnera-Amravati,India.
[2] Department of Mathematics, S.D.M.B. Science and Arts College, Shegaon, Dist-Buldana.
E-mail:vmete5622@gmail.com ,kailasmule@rediffmail.com,

Abstract: In the $f(R)$ theory of gravity, we have studied electromagnetic fields in Bianchi type-VIo space time by considering the general case. It is found that if the study is confined to the case of $f(R)=\mu R$. It is observed that the convergent and isotropic solution of the metric function can be evolved with the components of the vector potentials.

Keyword: Bianchi Type VIo, Electromagnetic Field, $f(R)$ theory of gravity, isotropy, constant vector potential.

1. Introduction

Basically, two kinds of alternative accelerated expansion of the universe have been proposed for this unexpected observational phenomenon. One is negative pressure known as dark energy (DE) which induces a late-time accelerating cosmic expansion. The other is the modified gravity, which originate from the idea that the general relativity is incorrect in the cosmic scale and therefore need to be modified. In order to explain the nature of the DE and accelerated expansion, a variety of theoretical models have been proposed in literature. There are several modified gravity theories like $f(R)$ gravity formulated by Nojiri and Odinstov [4,6]. The idea of introducing additional terms of the Ricci scalar to the Einstein-Hilbert action did not begin years ago with the $f(R)$ theory of gravity paper by Carroll *et.al* [5]. He explained the presence of a late time cosmic acceleration of the universe in $f(R)$ theory of gravity.

The $f(T)$ theory of gravitation formulated by Nojiri and Odinstov [3].To justify the current expansion of the universe come from modified or alternative theories of gravity, $f(T)$ Theory of gravity is one such example which has been recently developed. This theory is a generalized version of teleparallel gravity in which Weitzen block connection is used instead of Levi- Civita connection. The interesting feature of this theory is that it may explain the current acceleration without involving dark energy. In our opinion, one of interesting and prospective version of modified gravity theories is the $f(R,T)$ gravity proposed by Harko *et al* [8,9] and Myrzakulov [10]. In $f(R,T)$ theory of gravity, cosmic acceleration may result not only due to geometrical contribution to the total cosmic energy density but it is also depends on matter contents. Many authors have investigated different problem within the scope of $f(R,T)$ theory. Bijan Saha [11] has studied the interacting scalar and electromagnetic fields in Bianchi type I universe. Solanke and Karade [12] have studied electromagnetic field in $f(R,T)$ gravity in Bianchi type-III space-time. Our interest is to explore the role of scalar and electromagnetic field played in the amended $f(R,T)$ gravity in other Bianchi types or other metric universe. In this paper we consider axially symmetric metric universe.

2. Gravitational field equations of $f(R)$ gravity

The field equation of $f(R,T)$, theory due to Harko[8-9] are deduced by varying the action.

$$s=\int f(R,T)\sqrt{-g}\,d^4x+\int L_m\sqrt{-g}\,dx^4, \tag{2.1}$$

where L_m are Lagrangian and other symbols have their usual meaning.

Varying the action (1.1) with respect to g^{ij}, which yields as

$$\delta S = \frac{1}{2x}\int\left\{ f_R(R,T)\frac{\delta R}{\delta g^{ij}} + f_T(R,T)\frac{\delta T}{\delta g^{ij}} + \frac{\delta f(R,T)}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{ij}} + \frac{2x}{\sqrt{-g}}\frac{\delta\left(L_m\sqrt{-g}\right)}{\delta g^{ij}}\right\}\sqrt{-g}\,d^4x \quad , \tag{2.2}$$

Considering $\delta s = 0$ from equation (2.2) upon integration, we can obtain

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}[\nabla^i\nabla_j - \nabla_i\nabla_j\right)f_R(R,T) = xT_{ij} - f_T(R,T)\left[T_{ij} + \theta_{ij}\right] \quad , \tag{2.3}$$

where ∇_i is the constant derivative.

Replaced $f(R,T)$ by $f(R)$ in (2.3), we obtain

$$f_R(R)R_{ij} - \frac{1}{2}f(R)g_{ij} + \left(g_{ij}[\nabla^i\nabla_j - \nabla_i\nabla_j\right)f_R(R) = xT_{ij} \quad , \tag{2.4}$$

Taking trace of equation (2.4), we get

$$f_R(R)g^{ij}R_{ij} - \frac{1}{2}f(R)g^{ij}g_{ij} + \left(g^{ij}g_{ij}\nabla^i\nabla_j - g^{ij}\nabla_i\nabla_j\right)f_R(R) = xg^{ij}T_{ij} \quad ,$$

$$f_R(R)R - \frac{4}{2}f(R) + \left[4\nabla^i\nabla_j - \nabla^i\nabla_j\right]f(R)R = xT \quad ,$$

$$f_R(R)R - 2f(R) + 3\nabla^i\nabla_j f(R)R = xT \quad ,$$

$$3\nabla^i\nabla_j f_R(R) = xT + 2f(R) - F_R(R) \quad ,$$

$$\nabla^i\nabla_j f_R(R) = \frac{1}{3}xT + \frac{2}{3}f(R) - \frac{1}{3}F_R(R) \quad , \tag{2.5}$$

We consider the particular case $f(R) = \mu R$.

It follows notation

$$f(R) = \frac{\partial F(R)}{\partial R} = \frac{\partial}{\partial R}\mu R = \mu \quad , \tag{2.6}$$

The field (2.4) with the aid of (2.6), reduces to

$$\mu R_{ij} - \frac{1}{2}\mu R g_{ij} + \left(g^{ij}g_{ij}\nabla^i\nabla_j - g^{ij}\nabla_i\nabla_j\right)\mu = xT_{ij} \quad ,$$

$$\mu R_{ij} - \frac{1}{2}\mu R g_{ij} = xT_{ij} \quad , \tag{2.7}$$

The equation (2.5) with the aid of (2.6), reduces to

$$\nabla^i\nabla_j\mu = \frac{1}{3}xT + \frac{2}{3}(\mu R) - \frac{1}{3}\mu R \quad ,$$

$$\frac{1}{3}xT + \frac{1}{3}(\mu R) = 0 \quad ,$$

$$xT + (\mu R) = 0 \quad , \tag{2.8}$$

Using (2.8) in (2.7), we have

$$\mu R_{ij} - \frac{1}{2}(-xT)g_{ij} = xT_{ij} \quad ,$$

$$\mu R_{ij} + \frac{1}{2}(xT)g_{ij} = xT_{ij} \quad , \tag{2.9}$$

$$\mu R^i_j + \frac{1}{2}(xT)g^i_j = xT^i_j \quad ,$$

3. Energy momentum tensor for electromagnetic field

Energy momentum tensor for electromagnetic field is given by

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\,L_m\right)}{\delta g^{ij}} = -\frac{2}{\sqrt{-g}}\left(-\sqrt{-g}\,\frac{\delta L_m}{\delta g^{ij}} + L_m\frac{\delta\sqrt{-g}}{\delta g^{ij}}\right) = 2\frac{\delta L_m}{\delta g^{ij}} + \frac{2}{\sqrt{-g}}L_m\frac{\delta\sqrt{-g}}{\delta g^{ij}},$$

$$= 2\frac{\delta L_m}{\delta g^{ij}} + \frac{2}{\sqrt{-g}}L_m\left(\frac{-\frac{1}{2}\sqrt{-g}\;g_{mn}\delta g^{mn}}{\delta g^{ij}}\right) = 2\frac{\delta L_m}{\delta g^{ij}} - g_{mn}L_m\frac{\delta g^{mn}}{\delta g^{ij}} = 2\frac{\delta L_m}{\delta g^{ij}} - g_{mn}\delta_i^m\delta_i^n,$$

$$T_{ij} = 2\frac{\delta L_m}{\delta g^{ij}} - g_{ij}L_m = 2\frac{\delta L_m}{\delta g^{ij}} - L_m g_{ij}, \tag{3.1}$$

where $L_m = \frac{1}{4}F_{kl}F^{kl}$ (3.2)

where F_{kl} electromagnetic field.

$$\frac{\delta L_m}{\delta g^{ij}} = \frac{1}{4}\frac{\partial}{\partial g^{ij}}F_{kl}F^{kl},$$

$$\frac{\delta L_m}{\delta g^{ij}} = \frac{1}{4}\frac{\partial}{\partial g^{ij}}F_{kl}g^{\alpha k}g^{\beta l}F_{\alpha\beta},$$

$$\frac{\delta L_m}{\delta g^{ij}} = \frac{1}{2}g^{\alpha k}F_{\alpha i}F_{kj}, \tag{3.3}$$

Using (3.3), the equation (3.1) reduces to

$$T_{ij} = \frac{1}{4}F_{kl}F^{kl}g_{ij} - F_{ki}F_j^k, \tag{3.4}$$

4. The metric and field equations.

We consider the axially symmetric metric in the form

$$ds^2 = dt^2 - a_1^2dx^2 - a_2{}^2e^{-2m^2x}dy^2 - a_3{}^2e^{2m^2x}dz^2, \tag{4.1}$$

where a_1, a_2, and a_3, are functions of time t.

5. Electromagnetic field tensor F_{ij}

The electromagnetic field tensor is given by

$$F_{ij} = \frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}, \tag{5.1}$$

To achieve the capability with non-static space time (4.1), we assume electromagnetic vector potential in the form

$$A_i = \left[u(\alpha)v_1(t) \quad v_2(t),\ v_3(t)\ \ v_4(t)\ \right], \tag{5.2}$$

From (5.1) and (5.2) we can easily deduce

$$F_{14} = u\dot{v}_1,\ F_{24} = \dot{v}_2,\ F_{34} = \dot{v}_3, \tag{5.3a}$$

$$F^{41} = \frac{u\dot{v}_1}{a_1{}^2},\quad F^{42} = -\frac{\dot{v}_2}{a_2{}^2e^{-2m^2x}},\quad F^{43} = \frac{\dot{v}_3}{a_3{}^2e^{2m^2x}}, \tag{5.3b}$$

$$F_4^2 = -\frac{\dot{v}_2}{a_2{}^2e^{-2m^2x}},\quad F_4^3 = -\frac{\dot{v}_3}{a_3{}^2e^{2m^2x}}, \tag{5.3c}$$

$$F_1^4 = -u\dot{v}_1,\ F_2^4 = -\dot{v}_2,\ F_3^4 = -\dot{v}_2, \tag{5.3d}$$

From above equations we can compute

$$F_\mu F^\mu = -2\left[\frac{u\dot{v}_1^2}{a_1^2} + \frac{\dot{v}_2^2}{a_2 e^{-2m^2x}} + \frac{\dot{v}_3^2}{a_3^2 e^{2m^2x}}\right] \tag{5.4}$$

Using (3.4), we establish the following nonzero components of the energy momentum tensor of material field

$$T_1^1 = \frac{1}{2}\left[\frac{u^2\dot{v}_1^2}{a_1^2} - \frac{\dot{v}_2^2}{a_2^2 e^{-2m^2x}} - \frac{\dot{v}_3^2}{a_3^2 e^{-2m^2x}}\right], \tag{5.4a}$$

$$T_2^2 = \frac{1}{2}\left[\frac{u^2\dot{v}_1^2}{a_1^2} + \frac{\dot{v}_2^2}{a_2^2 e^{-2m^2x}} - \frac{\dot{v}_3^2}{a_3^2 e^{-2m^2x}}\right], \tag{5.4b}$$

$$T_3^3 = -\frac{1}{a_3^2 e^{2m^2x}}\left[\frac{a_3^2 u^2 \dot{v}_1^2}{2a_1^2} e^{2m^2x} + \frac{a^2{}_3 \dot{v}_2^2}{2a_2^2 e^{-2m^2x}} e^{2m^2x} - \frac{\dot{v}_3^2}{2}\right], \tag{5.4c}$$

$$T_4^4 = \frac{1}{2}\left[\frac{u^2\dot{v}_1^2}{a_1^2} + \frac{\dot{v}_2^2}{a_2^2 e^{-2m^2x}} + \frac{\dot{v}_3^2}{a_3^2 e^{-2m^2x}}\right], \tag{5.4d}$$

From equations (5.4a) (5.4b) (5.4c) and (5.4d), we can deduced the components of energy Tensor as follows

$$T_i^i = 0 \tag{5.5}$$

6. Maxwell's Equation

The Maxwell Equations are given by

$$\frac{(\dot{v}_1)^{\cdot}}{v_1} + \frac{\dot{v}_1^2}{v_1^2} + \frac{\dot{v}_1}{v_1}\left[\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1}\right] = 0, \tag{6.1a}$$

$$\frac{(\dot{v}_2)^{\cdot}}{v_2} + \frac{\dot{v}_2^2}{v_2^2} + \frac{\dot{v}_2}{v_2}\left[\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3} - \frac{\dot{a}_2}{a_2}\right] = 0, \tag{6.1b}$$

$$\frac{(\dot{v}_3)^{\cdot}}{v_3} + \frac{\dot{v}_3^2}{v_3^2} + \frac{\dot{v}_3}{v_3}\left[\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3}\right] = 0, \tag{6.1c}$$

$$u = c, \tag{6.1d}$$

where c is a constant.

7. Solution of vector potential

Considering the trivial solution component of Ricci tensor form field (2.9), we get

$$\frac{\dot{v}_1 \dot{v}_2}{v_1 v_2} = 0 \quad , \tag{7.1a}$$

$$\frac{\dot{v}_1 \dot{v}_3}{v_1 v_3} = 0 \quad , \tag{7.1b}$$

$$\frac{\dot{v}_2 \dot{v}_3}{v_2 v_3} = 0 \quad , \tag{7.1c}$$

From (6.1a to 6.1c),we write

$$\frac{\dot{v}_1 \dot{v}_2}{v_1 v_2} = \frac{\dot{v}_1 \dot{v}_3}{v_1 v_3} = \frac{\dot{v}_2 \dot{v}_3}{v_2 v_3} = 0 \quad , \tag{7.2}$$

which further imply

$$\frac{\dot{v}_1}{v_1} = \frac{\dot{v}_2}{v_2} = \frac{\dot{v}_3}{v_3} = \frac{\dot{D}}{D} , \tag{7.3}$$

where D is some unknown function of t.
Using (7.3), we obtain

$$v_1 = K_1 D \;\; v_2 = K_2 D \;\; v_3 = K_3 D , \tag{7.4}$$

where k's are constants of integration.

8. Solution of field equations.
As in Solanke and Karade (12), we consider

$$\frac{u^2 \dot{v}_1^2}{a_1^2} + \frac{\dot{v}_2^2}{a_2^2 e^{-2m^2 x}} + \frac{\dot{v}_3^2}{a_3^2 e^{2m^2 x}} = \left[\frac{u^2 v_1^2}{a_1^2} + \frac{v_2^2}{a_2^2 e^{-2m^2 x}} + \frac{v_3^2}{a_3^2} \right] \left(\frac{\dot{D}}{D} \right)^2 = -I \left(\frac{\dot{D}}{D} \right)^2$$

Now our plan is to express the components of T_j^i in terms of T_4^4

$$T_1^1 = \frac{u^2 \dot{v}_1^2}{2a_1^2} - \frac{\dot{v}_2^2}{2a_2^2 e^{-2m^2 x}} - \frac{\dot{v}_3^2}{2a_3^2 e^{2m^2 x}} = -T_4^4 - \frac{u^2 v_1^2}{a_1^2} I \left(\frac{\dot{D}}{D} \right)^2 , \tag{8.1a}$$

$$T_2^2 = -\frac{u^2 \dot{v}_1^2}{2a_1^2} + \frac{\dot{v}_2^2}{2a_2^2 e^{-2m^2 x}} - \frac{\dot{v}_3^2}{2a_3^2 e^{2m^2 x}} = -T_4^4 + \frac{v_2^2}{a_2^2 e^{-2m^2 x}} \left(\frac{\dot{D}}{D} \right)^2 , \tag{8.1b}$$

$$T_3^3 = -\frac{u^2 \dot{v}_1^2}{2a_1^2} - \frac{\dot{v}_2^2}{2a_2^2 e^{-2m^2 x}} + \frac{\dot{v}_3^2}{2a_3^2 e^{2m^2 x}} = -T_4^4 + \frac{v_3^2}{a_3^2 e^{-2m^2 x}} \left(\frac{\dot{D}}{D} \right)^2 , \tag{8.1C}$$

$$T_4^4 = \frac{u^2 \dot{v}_1^2}{2a_1^2} + \frac{\dot{v}_2^2}{2a_2^2 e^{-2m^2 x}} + \frac{\dot{v}_3^2}{2a_3^2 e^{2m^2 x}} = -\frac{1}{2} I \left(\frac{\dot{D}}{D} \right)^2 , \tag{8.1d}$$

By using (7.4), we get trace of energy momentum tensor as

$$T = I\left(\frac{\dot{D}}{D}\right)^2 - \frac{1}{2}I\left(\frac{\dot{D}}{D}\right)^2 = 0 \quad , \tag{8.2}$$

Considering the non-trivial components of Ricci tensor from field equation (2.9)

$$\mu\, R_j^i + \frac{x}{2}\, xTg_j^i = xT_j^i \quad , \tag{8.3}$$

$$\mu\left[-\frac{2m^4}{a^2{}_1} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3}\right] = x\left[-T_4^4 + \frac{u^2v_1^2}{a_1^2}\left(\frac{\dot{D}}{D}\right)^2\right] , \tag{8.3a}$$

$$\mu\left[\frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3}\right] = x\left[-T_4^4 + \frac{v_2^2}{a_2^2e^{-2m^2x}}\left(\frac{\dot{D}}{D}\right)^2\right] , \tag{8.3b}$$

$$\mu\left[\frac{\ddot{a}_3}{a3} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3}\right] = x\left[-T_4^4 + \frac{v_3^2}{a_3^2e^{2m^2x}}\left(\frac{\dot{D}}{D}\right)^2\right] , \tag{8.3c}$$

By using (7.4), above equations leads to

$$-\frac{2m^4}{a^2{}_1} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} = 0 \quad , \tag{8.4a}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} = 0 \quad , \tag{8.4b}$$

$$\frac{\ddot{a}_3}{a3} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} = 0 \quad , \tag{8.4c}$$

With the help of (7.3), we can write the equation (6.1) as

$$\left(\frac{\dot{D}}{D}\right)^{\cdot} + \left(\frac{\dot{D}}{D}\right)^2 + \frac{\dot{D}}{D}\left[\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1}\right] = 0 \quad , \tag{8.4d}$$

$$\left(\frac{\dot{D}}{D}\right)^{\cdot} + \left(\frac{\dot{D}}{D}\right)^2 + \frac{\dot{D}}{D}\left[\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3} - \frac{\dot{a}_2}{a_2}\right] = 0 \quad , \tag{8.4e}$$

$$\left(\frac{\dot{D}}{D}\right)^{\cdot} + \left(\frac{\dot{D}}{D}\right)^2 + \frac{\dot{D}}{D}\left[\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a3}\right] = 0 \quad , \tag{8.4f}$$

This equation further imply that

$$\frac{\dot{a}_1}{a_1} = \frac{\dot{a}_2}{a_2} = \frac{\dot{a}_3}{a_3}, \tag{8.5}$$

whichon Integration with respect to t, we get

$$a_1 = K_4 a_2 \quad a_2 = K_5 a_3 \quad a_3 = K_6 a_1, \tag{8.6}$$

where k's are constants of integration.
From (8.4) and (8.5), we get

$$\frac{\ddot{a}_2}{a_2} + 2\left(\frac{\dot{a}_2}{a_2}\right)^2 = 0, \tag{8.7a}$$

$$\frac{\ddot{a}_3}{a_3} + 2\left(\frac{\dot{a}_3}{a_3}\right)^2 = 0, \tag{8.7b}$$

Upon integration, we get

$$a_2 = (3k_7 t + 3k_8)^{\frac{1}{3}}, \tag{8.8a}$$

$$a_3 = (3k_9 t + 3k_{10})^{\frac{1}{3}}, \tag{8.8b}$$

With the help of (8.6) and (8.8a), we have

$$a_1 = (3k_{11} t + 3k_{12})^{\frac{1}{3}}. \tag{8.8c}$$

By using the equations (8.5), (8.8a), (8.8b) and (8.8c) we get

$$k_7 = K_9 = k_{11} = d_1 \quad k_8 = K_{10} = k_{12} = d_2$$

$$a_1 = a_2 = a_3 = (3d_1 t + 3kd_2)^{\frac{1}{3}} \tag{8.9}$$

$$\frac{\dot{D}}{D} = 0, \quad \dot{D} = 0, \quad D = C, \tag{8.10}$$

where C is constant.
From equation (8.9) and (8.4a) we get
$m = o$
Using equations (8.9), the line element (4.1) reduces to

$$ds^2 = dt^2 - (3d_1 t + 3kd_2)^{\frac{2}{3}}\left[dx^2 + e^{-2m^2 x} dy^2 + e^{2m^2 x} dz^2\right]. \tag{8.11}$$

Conclusion

In this paper, we have considered the particular case $f(R) = \mu R$ model in Bianchi type VIo.

1) It is observed that convergent non-singular, isotropic solution can be evolved for the metric function and the components of vector potential.
2) Model shows that universe expand algebraically in $f(R) = \mu R$ theory of gravity.
3) The metric function in non-static space time admits constant value at early time of the universe. T tends to zero and after that the metric function start increasing with increase in cosmic time and finally diverge to infinity as time tend to infinity .This shows that the universe expand and approaches to infinite volume.
4) It is also interesting to note that the investigated model is free from singularity. Hence the model approaches isotropic for the anytime.

References

[1] M.A. Melvin ,(1975), "*Homogeneous axial cosmologies with electromagnetic field and dust*", Ann. NewyorkAcad. Sci.; **262**: 253-274.

[2] R. Maartens,(2000)., "*Cosmological magnetic fields*". Pramana. J. Phys.; **55**: 575-58315.

[3] D. Grasso , H.R.Rubinstein ,(2001), "*Magnetic fields in the early universe*", Phys. Rep.; **348**: 163-266.

[4] S.Nojiri and S. D. Odintsov(2003), "Modified gravity with negative and positive power of curvature :Unification of inflation and cosmic acceleration phys.Rev D 68,123512

[5] S. M. Carroll, V. Duvvuri and M. S. Turner, (2004). Phys. Rev. D 70, 043528

[6] S. Nojiri and S. D. Odintsov, "Introduction to modified gravity and gravitational alternative for daek energy , arXiv:hep-th/060123V5 (2006).

[7] S. Nojiri, S. D. Odintsov and P. V. Tretyakov, (2007); Phys. Lett. B 651, 224

[8] T. Harko, T. S. Koivisto and F. S. N. Lobo,(2010)., arXiv:1007.4415

[9] T. Harko and F. S. N. Lobo, (2010). Eur. Phys. J. C 70, 373

[10] Myrazakulov,(2012) "RFW Cosmologicaly in f(R,T) gravity".The E.ph.J. 72:2203,

[11] BijanSaha (2015)."Introduction Scalar and Electromagnetic Field in f(R.T) theory of gravity" Int. j.of Phy..1073-75,31.

[12] D.T.Solanke,T.M.Karade,(2016),prespacetime j.,Nov,2016,vol.7,issue13,pp:1766-1785