



## AN ANALYTICAL SOLUTION OF TIME FRACTIONAL ORDER HYGROTHERMOELASTIC RESPONSE OF CYLINDER BY RAMP TYPE HEATING

Nagesh Dhore<sup>1</sup> and Lalsingh Khalsa<sup>2</sup>

<sup>1,2</sup>Department of Mathematics, M.G. College, Armori, Gadchiroli, India  
Corresponding Email: nageshdhore@gmail.com, lalsinghkhalsa@yahoo.com

Communicated : 18.01.2023

Revision : 22.02.2023 & 07.03.2023  
Accepted : 29.03.2023

Published : 30.05.2023

### ABSTRACT:

In the context of fractional calculus, an analytical approach is suggested for resolving the coupled time-fractional order hygrothermoelastic equations for a non-simple cylinder. The transient response of an infinitely long cylinder subjected to ramp-type hygrothermal loadings at the surface is examined using the theory of fractional hygrothermoelasticity. Closed-form expressions for temperature and moisture have been derived in the Laplace domain using the Laplace transform method and introducing a new auxiliary function. Finally using the Riemann-sum approximation method, the solution is obtained in a time domain. The numerical findings of the transient response of the hygrothermoelastic fields are provided graphically to show the effect of the ramping time parameter inside the hygrothermal field.

**Keywords :-** Hygrothermoelasticity, Non-Simple Nano-Cylinder, Laplace Transform, Ramp Type Heating, Fractional Order Differential Equation.

### INTRODUCTION :

During initial manufacture, most buildings, particularly those made of porous composite materials, are frequently exposed to varying environmental factors including temperature and moisture. For a large range of materials, temperature and moisture are not independent of one another but rather are symbiotic. Determining how temperature and moisture affect these material's and structure's stresses and deformation is therefore of major interest. Many researchers have looked into the related hygrothermoelastic issues.

A. Kilbas, H. M. Srivastava, and J. J. Trujillo [1] investigated sequential and non-sequential fractional order linear differential equations as well as systems of linear fractional differential equations connected to the Riemann-Liouville and Caputo derivatives. A. Chaves [2] proposed a diffusion equation for fractional derivatives that produces the Levy statistics. In his hygrothermomechanical bending analysis of

variable-thickness thin rectangular plates, A.M. Zenkour clamps two of the plate's opposite edges while leaving the other two opposite edges unsupported [3]. In order to understand how the presence of two unique temperatures permits dependency on higher gradients, Chen, P.J., Gurtin, M.E., and Willams, W.O. turn to a theory that incorporates mechanical effects [4]. In light of a fresh analysis of heat conduction with fractional order, H. M. Youssef developed a novel thermoelasticity theory model, and its uniqueness theorem has been confirmed [7]. The characterization of the quality factor resulting from the static prestress in the traditional Caputo and Caputo-Fabrizio fractional thermoelastic silicon microbeam is studied by H.M. Youssef, A.A. El-Bary, and E.A.N. Al-Lehaibi [8]. Povstenko [10] uses the time-fractional diffusion equation to describe the radial diffusion in a cylinder with radius R. In a multilayer plate subjected to hygrothermal loadings at the external surfaces, R. Chiba and

Y. Sugano studied the dispersion of transitory heat and moisture and the ensuing hygrothermal stress field [11]. By substituting a fractional derivative of order  $\beta$  for the first-order time derivative in the classic diffusion equation, R. Gorenflo, F. Mainardi, D. Moretti, and P. Paradisi were able to derive the time fractional diffusion equation  $\beta \in (0, 1)$  [12]. R. Lifshitz and M.L. Roukes [13] highlight the significance of thermoelastic damping as a fundamental dissipation process for small-scale mechanical systems in light of current efforts to construct high-Q micrometer- and nanometer-scale electromechanical systems. In order to design a new model of two-temperature hygrothermoelastic diffusion theory for a non-simple rigid material, Sonal Bhoyar, Vinod Varghese, and Lalsingh Khalsa researched the theoretical framework to combine both the traditional Fourier's and Fick's laws [14]. The Special Functions of Fractional Calculus were described by V. Kiryakova, and they were crucial in the development of control systems, sophisticated mathematical models of various physical, chemical, economic, management, and bioengineering phenomena, as well as solutions to fractional order (or multi-order) differential and integral equations. A linear theory of linked heat and moisture is used by W.J. Chang, T.C. Chen, and C.I. Weng [16] to investigate the transient reactions in an infinitely long annular cylinder subjected to hygrothermal loadings. Within the context of fractional calculus, X.Y. Zhang developed and investigated many models of hygrothermoelastic theory related to relaxation times or phase lag in [17–22]. These models were based on fraction diffusion wave theory. Y. Povstenko [24] evaluated the non-axisymmetric solutions to the time-fractional diffusion-wave equation in an infinite cylinder. The time-fractional diffusion-wave equation is taken into account in an infinite cylinder for the case of three spatial variables  $r$ ,  $\phi$  and  $z$ . In a

context of fractional Order Thermoelastic Waves, Youssef, Hamdy, Elsibai, Khaled, and El-Bary investigated a mathematical model of cylindrical nanobeam [25].

### **Formulation for Time fractional Hygrothermal Equation For Non Simple Medium:**

In this study, we first suggest a theory of time fractional hygro-thermoelasticity theory for a non-simple medium. For the sake of simplicity, it is assumed that heat and moisture are coupled and that both affect the medium's elastic stresses. Conversely, elastic deformation has no impact on them. As a result, the interaction between heat and moisture can be represented as the diffusion of water vapour through a material's pores, which are partially filled with solids and partially with air. In general, according to [16], the amount of moisture absorbed by a unit mass of a solid,  $M$ , can be assumed to depend linearly on the concentration of water vapour contained in a unit volume of void,  $C$ , and the temperature,  $T$ , and change in moisture and temperature is confined within a small range, the amount of moisture absorbed by a unit mass of a solid is given by,

$$M = \chi C - \omega T + \text{constant} \quad (1)$$

where  $\chi$  and  $\omega$  are material constants. Then the amount of moisture contained in the composite per unit mass of solid,  $m$ , can be expressed as

$$m = \frac{\nu' C}{\rho} + M \quad (2)$$

where  $\nu'$  is the volume fraction of the voids, and  $\rho$  is the density of the material,  $\rho = (1 - \nu') \rho_s$ ,  $\rho_s$  being the density of the solid without voids. Due to the principles of energy conservation and mass conservation, we can write

$$\nabla q_h = \rho\gamma \frac{\partial M}{\partial t} - \rho c_p \frac{\partial T}{\partial t} \tag{3}$$

$$\nabla q_m = -\frac{\rho}{v'} \frac{\partial M}{\partial t} - \frac{\partial C}{\partial t} \tag{4}$$

where  $\gamma$  is the amount of heat released from per unit mass of moisture and  $c_p$  is the specific heat at constant pressure.

The assumption is made in accordance with [25] that heat and moisture obey time-fractional Fourier and Fick's laws, where the matter flux has the power of a time-nonlocal kernel characterising "long-tale" memory. Thus, the heat flux vector  $q_h$  and moisture flux vector  $q_m$  take the following forms, respectively,

$$q_h(t) = \begin{cases} -\frac{D_h}{\Gamma(\alpha)} \frac{\partial}{\partial t} \int_0^t (t-\tau)^{\alpha-1} \nabla T(\tau) d\tau, & 0 < \alpha \leq 1 \\ -\frac{D_h}{\Gamma(\alpha-1)} \int_0^t (t-\tau)^{\alpha-2} \nabla T(\tau) d\tau, & 1 < \alpha \leq 2 \end{cases} \tag{5}$$

$$q_m(t) = \begin{cases} -\frac{D_m}{\Gamma(\beta)} \frac{\partial}{\partial t} \int_0^t (t-\tau)^{\beta-1} \nabla C(\tau) d\tau, & 0 < \beta \leq 1 \\ -\frac{D_m}{\Gamma(\beta-1)} \int_0^t (t-\tau)^{\beta-2} \nabla C(\tau) d\tau, & 1 < \beta \leq 2 \end{cases} \tag{6}$$

in which  $q_h$  and  $q_m$  represent the coefficients of heat conduction and moisture diffusion,  $\alpha, \beta$  are the fractional orders, respectively, and  $\Gamma(*)$  is the Gamma function.

Substituting Eqs. (5) and (6) into Eqs.(3) and (4), we get

$$D_h \nabla^2 T = \rho c_p \frac{\partial^\alpha T}{\partial t^\alpha} - \rho\gamma \frac{\partial^\alpha M}{\partial t^\alpha} \quad 0 < \alpha \leq 2 \tag{7}$$

$$D_m \nabla^2 C = \frac{\partial^\beta C}{\partial t^\beta} - \frac{\rho}{v'} \frac{\partial^\beta M}{\partial t^\beta} \quad 0 < \beta \leq 2 \tag{8}$$

in which  $(\partial^\alpha / \partial t^\alpha)$  and  $(\partial^\beta / \partial t^\beta)$  is the Caputo fractional derivatives,  $\alpha, \beta$  represents the fractional order of a Caputo fractional

derivative with respect to time  $t$ ,  $\Gamma(*)$  is the Gamma function, and Caputo fractional derivative is written as[3]

$$\frac{d^\alpha f(t)}{dt^\alpha} = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} \frac{d^n f(\tau)}{d\tau^n} d\tau, & n-1 < \alpha < n \\ \frac{d^n f(\tau)}{d\tau^n}, & \alpha = n \end{cases} \tag{8}$$

The quantity M in Eqs. (7) and (8) can be eliminated using Eq. (1). Substitute value of M in Eqs. (7) and (8), we get system of linearly coupled partial differential equation of moisture(C) and temperature(T) as follow,

$$D \nabla^2 T = \frac{\partial^\alpha T}{\partial t^\alpha} - \eta \frac{\partial^\alpha C}{\partial t^\alpha}, \quad 0 < \alpha \leq 2 \tag{9}$$

$$D \nabla^2 C = \frac{\partial^\beta C}{\partial t^\beta} - \lambda \frac{\partial^\beta T}{\partial t^\beta}, \quad 0 < \beta \leq 2 \tag{10}$$

Where  $D = \frac{D_h}{\rho(c_p + \gamma\omega)}$ ,  $\eta = \frac{\gamma\chi}{c_p + \gamma\omega}$ ,  $D = \frac{D_m v'}{v' + \rho\chi}$ ,  $\lambda = \frac{\rho\omega}{v' + \rho\chi}$  \tag{11}

The two-temperature model is a non-classical thermoelasticity theory of elastic solids that is currently being introduced. In this context [4] suggested classifying real materials into simple and non-simple materials by taking into account two temperatures, conductive and thermodynamic and they have shown that the two temperatures are related by,  $\phi = T - b \nabla^2 T$ ,  $b > 0$  in which  $\phi$  is the thermodynamic temperature,  $T$  is the conductive temperature, and  $b$  is the temperature discrepancy factor. Thus the thermodynamics and conductive temperatures are not identical for non-simple materials while they are identical for simple materials.

The material parameter b is a crucial distinction between the two-temperature thermoelasticity theory and the classical theory.

Specifically, in the limit as  $b \rightarrow 0$ ,  $\phi \rightarrow T$  which gives rise to the classical theory, i.e. one-temperature generalized thermoelasticity theory. Therefore, for a non-simple medium, Eq. (9) can be written as,

$$D\left(1 + \frac{b}{\kappa} \frac{\partial}{\partial t}\right) \nabla^2 T = \frac{\partial^\alpha T}{\partial t^\alpha} - \eta \frac{\partial^\alpha C}{\partial t^\alpha} \quad 0 < \alpha \leq 2 \quad (12)$$

### Formulation of the problem

The cylindrical coordinates system  $(r, \psi, z)$  is utilised to examine the time-fractional order hygrothermoelastic response of a cylindrical ramp-type heating structure with circular cross sections that is exposed to axisymmetric hygrothermal loadings at the surface, as illustrated in Figure 1.

Take a look at the tiny flexural deflections of a thin elastic cylinder with  $z$  axes specified along the longitudinal with length  $z(-h/2 \leq z \leq h/2)$ , width radius  $r(0 \leq r \leq a)$  and  $\psi(0 \leq \psi \leq 2\pi)$ . When the beam is in equilibrium, it is unstretched, unstrained and has no damping mechanism, and the temperature is  $T_0$  everywhere.[26]

As previously stated, we simply take into account the hygrothermal effect on elastic stresses and deformation here. However, as a result of an elastic field, temperature (T) and moisture (C) do not vary. Due to the asymmetry of this problem, the coupled partial differential equation for a non-simple medium is represented by Eq. (10) and (12) subjected to the initial and boundary conditions

$$T(r, 0) = 0 \text{ and } C(r, 0) = 0 \quad 0 < r < 1 \quad (13)$$

$$\frac{\partial^\alpha T(r, 0)}{\partial t^\alpha} = 0 \text{ and } \frac{\partial^\beta C(r, 0)}{\partial t^\beta} = 0, \text{ for } 0 < r < 1, \text{ if } 1 < \alpha, \beta \leq 2$$

$$(14) \quad T|_{z=0} = \begin{cases} T_1 t & \text{for } 0 \leq t \leq t_0, \\ t_0 & \text{for } t \geq t_0, \end{cases} \quad T|_{z=h} = 0, \quad (15)$$

$$C|_{z=0} = \begin{cases} C_1 t & \text{for } 0 \leq t \leq t_0, \\ t_0 & \text{for } t \geq t_0, \end{cases}, \quad C|_{z=a} = 0, \quad (16)$$

where  $t_0$  is a ramp-type parameter, and  $T_1$  and  $C_1$  are both fixed constants and  $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$  is the Laplacian operator.

### Solution of the problem

Applying the Laplace transform to Eq. (10)- (16) defined by the formula

$$\bar{f}(s) = L[f(t)] = \int_0^\infty f(t) e^{-st} dt, \quad s > 0 \quad (17)$$

where  $s$  is the Laplace parameter

Then Eq. (10) - (16) takes form in Laplace domain as,

$$D\left(1 + \frac{b}{\kappa} s\right) \nabla^2 \bar{T} = s^\alpha (\bar{T} - \eta \bar{C}) \quad (18)$$

$$D \nabla^2 \bar{C} = s^\beta (\bar{C} - \lambda \bar{T}) \quad (19)$$

$$\bar{\theta}|_{x=0} = \frac{\theta_1}{t_0} \left( \frac{1 - e^{-t_0 s}}{s^2} \right) = G_1(s), \quad \bar{\theta}|_{x=a} = 0 \quad (20)$$

$$\bar{\vartheta}|_{x=0} = \frac{\vartheta_1}{t_0} \left( \frac{1 - e^{-t_0 s}}{s^2} \right) = G_2(s), \quad \bar{\vartheta}|_{x=a} = 0 \quad (21)$$

Assume that temperature and moisture are expressible in terms of an auxillary function F as follows,

$$\bar{T} = (D \nabla^2 - s^\beta) \bar{F} \quad (22)$$

$$\bar{C} = -s^\beta \lambda \bar{F} \quad (23)$$

If the newly introduced unknown function F satisfies the single equation as given below, then Eqs. (18) and (19) are automatically satisfied.

$$DL\left(1+\frac{bs}{\kappa}\right)\nabla^4\bar{F}-\left(Ds^\alpha+Ls^\beta+\frac{bLs^{1+\beta}}{\kappa}\right)\nabla^2\bar{F}+(1-\eta\lambda)s^{\alpha+\beta}\bar{F}=0 \quad (24)$$

Note that there are no thermal gradients in the  $r$  direction and that thermal gradients in the plane of the cross section along the  $z$  direction are significantly larger than gradients along the beam axis we neglect the terms  $\partial^2/\partial r^2$  and  $\partial/\partial r$  and replace  $\nabla^2$  by  $\partial^2/\partial z^2$  [13]. Then the general solution of Eq. (24) is given by,

$$\bar{F}=C_1e^{k_1z}+C_2e^{-k_1z}+C_3e^{k_2z}+C_4e^{-k_2z} \quad (25)$$

$$\text{Where, } k_1=\sqrt{p_1-\sqrt{p_1^2-4P_2}}, \quad k_2=\sqrt{p_1+\sqrt{p_1^2-4P_2}} \quad (26)$$

$$p_1=\frac{bs^{1+\beta}}{2D(bs+\kappa)}+\frac{s^\alpha\kappa}{2L(bs+\kappa)}+\frac{s^\beta\kappa}{2D(bs+\kappa)},$$

$$P_2=\frac{s^{\alpha+\beta}\kappa}{DL(bs+\kappa)}-\frac{s^{\alpha+\beta}\eta\lambda\kappa}{DL(bs+\kappa)} \quad (27)$$

and  $C_1, C_2, C_3, C_4$  are unknown arbitrary constants which are to be determined by boundary condition Eq.(20) and (21). Using Eq. (25) we can write temperature and moisture distribution in Laplace domain as

$$\bar{T}=(Dk_1^2-s^\beta)(C_1e^{z k_1}+C_2e^{-z k_1})+(Dk_2^2-s^\beta)(C_3e^{z k_2}+C_4e^{-z k_2}) \quad (28)$$

$$\bar{C}=-s^\beta\lambda(C_1e^{z k_1}+C_2e^{-z k_1}+C_3e^{z k_2}+C_4e^{-z k_2}) \quad (29)$$

Eq.(28) and (29) gives the temperature and moisture distribution of a non-simple nano-cylinder in Laplace domain.

### Numerical Inversion of the Laplace Transform

The numerical results and solutions in the time domain are determined using the Riemann-sum approximation approach [5]. Any function that exists in the Laplace domain ( $s$ -domain) can be translated to the time domain ( $t$ -domain) using this numerical method as follows,

$$f(t)=\frac{e^{kt}}{t}\left[\frac{1}{2}f(k)+\text{Re}\sum_{n=1}^N(-1)^n f\left(k+\frac{in\pi}{t}\right)\right] \quad (30)$$

Numerous computational studies have demonstrated that the value of  $k$  meets the relation  $kt \approx 4.7$  for faster convergence.

### Numerical Result and Discussion

In this section, we introduce the non-dimensional variables indicated below for simplification in order to achieve the numerical results [17].

$$\bar{x}=x/a, \quad \bar{z}=z/h, \quad \bar{h}=h/a, \quad \bar{w}=w/h, \quad \theta=\frac{T-T_0}{T_0}, \quad \theta=\frac{C-C_0}{\psi T_0},$$

$$t=(c/a)t, \quad c^2=E/\rho, \quad \sigma'_x=\sigma_x/E \quad (31)$$

For a porous composite material with material parameters, the hygrothermoelastic distribution of a beam is computed numerically [6].

$$\alpha_1=31.3\times 10^{-6}\text{ cm}/(\text{cm}^\circ\text{C}), \quad \alpha_2=2.68\times 10^{-3}\text{ cm}/(\text{cm}\% \text{H}_2\text{O}),$$

$$\varphi=0.5\text{ cm}^3/\text{g}, \quad \psi=0.5\text{ g}/(\text{cm}^\circ\text{C}), \quad D=2.16\times 10^{-5}\text{ m}^2/\text{s},$$

$$D=2.16\times 10^{-6}\text{ m}^2/\text{s}, \quad \rho=1590\text{ kg}/\text{m}^3, \quad E=64.3\text{ GPa}, \quad \nu=0.33. \quad (32)$$

The figures were prepared by using the non-dimensional variables defined in Eq. (31) for a wide range of beam length when  $a=1$ ,  $z=h/6$  and  $t=0.15$ .

From figure 2, the dimensionless temperature (also known as the thermodynamic temperature and the conductive temperature) for thermal and moisture diffusion behaviour is always larger when the hygrothermal coupling is taken into account along the time direction at different places of  $\bar{z}$ .

Figure 3 shows the variation in the thermodynamic and conductive temperature for various locations in the points along the radial direction, which depends on the time and space coordinate and the two-temperature parameter  $b$ . The value of  $b=0$  indicates the one-temperature theory, while  $b\neq 0$  indicates the two-temperature theory. The variation in the thermodynamic and conductive temperature along the radial direction may be due to the available sectional heat source. The plate's temperature reaches its highest point at  $x=0$ ,

and while it is maintained at 0 degrees Celsius, the temperature gradually decreases as it moves toward  $x=b$ . The findings are consistent with the previously [3] obtained information.

#### CONCLUSION :

In order to resolve the issue of coupled temperature and moisture distribution, this paper provides a novel analytical strategy that uses the Laplace transform technique and a transformation function. In light of this, the approach is suggested for analytically resolving issues involving coupled temperature and moisture transport in non-simple materials. The conclusion drawn as follows:

- Temperature distribution and moisture diffusion are significantly impacted by the fractional order parameter.

On the temperature and moisture distribution, time has a major impact.

#### REFERENCES:

- A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, Theory and Applications of Fractional Differential Equations, Elsevier Science Limited, Amsterdam, 2006.
- A. Chaves, A fractional diffusion equation to describe Levy flights, Phys. Lett. A, vol. 239, pp. 13–16, 1998.
- A.M. Zenkour, Bending of thin rectangular plates with variable-thickness in a hygrothermal environment, Thin-Walled Struct. 123 (2018) 333–340.
- Chen, P.J., Gurtin, M.E. and Willams, W.O. (1969) On the thermodynamics of non-simple elastic material with two temperatures, Zeitschrift für Angew Mathematik und Physik, 20, 107-112. <https://doi.org/10.1007/BF01591120>
- D. Tzou, Macro-to-micro heat transfer, Taylor & Francis, Washington DC, 1996. DOI: 10.1002/9781118818275
- G.C. Sih, J.G. Michopoulos and S.C. Chou, Hygrothermoelasticity, Springer Dordrecht, 1986. DOI: 10.1007/978-94-009-4418-3
- H. M. Youssef, Theory of fractional order generalized thermoelasticity, J. Heat Transfer (ASME), Vol. 6, 2010, DOI:10.1115/1.4000705.
- H.M. Youssef, A.A. El-Bary, E.A.N. Al-Lehaibi, Characterisation of the quality factor due to the static prestress in classical Caputo and Caputo–Fabrizio fractional thermoelastic silicon microbeam, Polym. J., Vol. 13, No. 1, pp. 27, 2020. DOI: 10.3390/polym13010027
- L. Debnath, D. Bhatta, Integral transforms and their applications, Chapman and Hall/CRC, New York, 2006. 10.1201/9781420010916
- Povstenko, Fractional radial diffusion in a cylinder, J. Mol. Liq., vol. 137, pp. 46–50, 2008.
- R. Chiba and Y. Sugano, Transient hygrothermoelastic analysis of layered plates with one-dimensional temperature and moisture variations through the thickness, Compos. Struct., Vol. 93, pp. 2260–2268, 2011. <https://doi.org/10.1016/j.compstruct.2011.03.014>
- R. Gorenflo, F. Mainardi, D. Moretti, and P. Paradisi, Time fractional diffusion: A discrete random walk approach, Nonlinear Dyn., vol. 29, pp. 129–143, 2002.
- R. Lifshitz and M.L. Roukes, Thermoelastic damping in micro- and nanomechanical systems, Phys. Rev. B, Vol. 61, No. 8, pp. 5600-5609, 2000. DOI: 10.1103/PhysRevB.61.5600
- Sonal Bhojar, Vinod Varghese & Lalsingh Khalsa, Hygrothermoelastic response in the bending analysis of elliptic plate due to hygrothermal loading, J. Therm. Stress., Vol. 43, No. 3, pp. 372-400,



2020. DOI: 10.1080/01495739.2019.1711477
- V. Kiryakova, The Multi-Index Mittag-Leffler Functions as an Important Class of Special Functions of Fractional Calculus, *Comput. Math. Appl.*, vol. 59, pp. 1885–1895, 2010.
- W.J. Chang, T.C. Chen, C.I. Weng, Transient hygrothermal stresses in an infinitely long annular cylinder: coupling of heat and moisture, *J. Therm. Stress.*, Vol. 14, No. 4, pp. 439-454, 1991. DOI: 10.1080/01495739108927078
- X.Y. Zhang and X.F. Li, Hygrothermoelastic damping of beam resonators with non-Fourier and non-Fick effects, *Thin-Walled Struct.*, Vol. 168, pp. 108283, 2021. DOI: 10.1016/j.tws.2021.108283
- X.Y. Zhang and X.F. Li, Transient response of a hygrothermoelastic cylinder based on fractional diffusion wave theory, *J. Therm. Stresses*, vol. 40, no. 12, pp. 1575-1594, 2017. DOI: 10.1080/01495739.2017.1344111
- X.Y. Zhang, Y. Peng and X.F. Li, Time-fractional hygrothermoelastic problem for a sphere subjected to heat and moisture flux, *J. Heat Transf.*, vol. 140, no. 122, pp. 1220021-122009, 2018. <https://doi.org/10.1115/1.4041419>
- X.Y. Zhang, Y. Peng, Y.J. Xie, Hygrothermoelastic response of a hollow cylinder based on a coupled time-fractional heat and moisture transfer model, *Z. Angew. Math. Phys.*, vol. 70, no. 2, pp. 1-21, 2019. <https://doi.org/10.1007/s00033-018-1047-1>.
- X.Y. Zhang, Y. Peng, Y.J. Xie, Hygrothermoelastic response of a hollow cylinder based on a coupled time-fractional heat and moisture transfer model, *Z. Angew. Math. Phys.*, Vol. 70, No. 2, pp. 1-21, 2019. <https://doi.org/10.1007/s00033-018-1047-1>
- Y. Peng, X.Y. Zhang, X.F. Li, Transient hygrothermoelastic response in a porous cylinder subjected to ramp-type heat-moisture loading, *J. Therm. Stress.*, Vol. 42, No. 12, pp. 1499–1514, 2019. DOI: 10.1080/01495739.2019.1653801
- Y. Povstenko, *Fractional Thermoelasticity*, Vol. 219, Springer, 2015. DOI: 10.1007/978-3-319-15335-3
- Y. Povstenko, Non-axisymmetric Solutions to time-fractional diffusion-wave equation in an infinite cylinder, *Fract. Calcul. Appl. Anal.*, vol. 14, pp. 418–435, 2011. DOI: 10.2478/s13540-011-0026-4
- Youssef, Hamdy & Elsibai, Khaled & El-Bary, A.. (2013). Fractional Order Thermoelastic Waves of Cylindrical Gold Nano-Beam. *ASME International Mechanical Engineering Congress and Exposition, Proceedings (IMECE)*. 8. DOI: 10.1115/IMECE2013-62876.
- E. Hosseinian, P.O. Theillet, O.N. Pierron, Temperature and humidity effects on the quality factor of a silicon lateral rotary micro-resonator in atmospheric air, *Sens. Actuator A Phys.*, Vol. 189, pp. 380–389, 2013. DOI: 10.1016/j.sna.2012.09.020

Figure 1

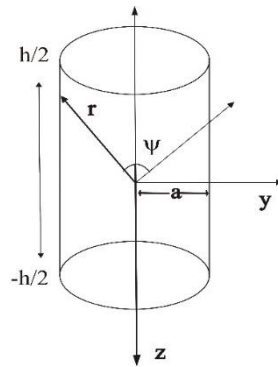


Figure 2: The temperature and moisture distribution along  $\bar{x}$  for various  $\bar{z}$

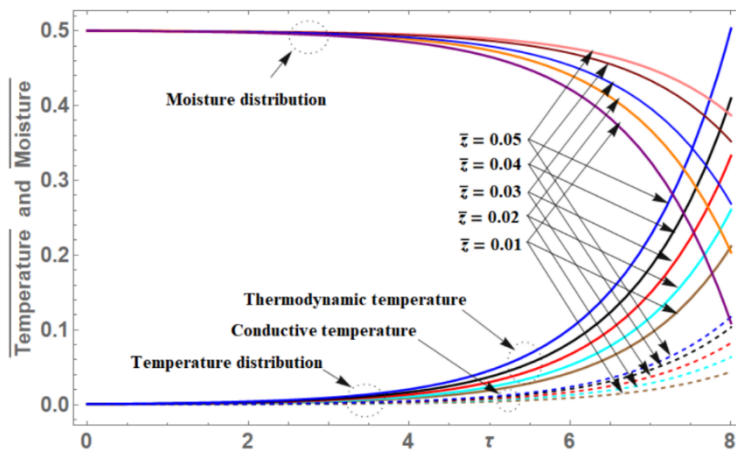


Figure 3: The temperature and moisture distribution along  $\bar{x}$  for various  $\tau$

