



GENERALIZATION AND CONVENTIONS OF VARIOUS COORDINATE SYSTEM

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Abstract:

In this paper, discuss boundary value problems coordinate systems like spherical coordinate systems, Cartesian coordinate system, generalization and conversion of system, unique coordinate system. Existence and uniqueness of solution, exponential smoothing for models. Periodic boundary problems requires the unit cell to be shape that will tile perfectly into three dimensional crystal. A common alternative that requires less volume is the truncated octahedron.

Keywords:

Boundary value problems, Cartesian coordinate spherical coordinate, smoothing. Periodic boundary problems etc.

Introduction:

In mathematics in the field of differential equations, a boundary value problem is a differential equation together with a set of additional restraints called the boundary condition. A solution to a boundary value problem is a solution to the differential equation which also satisfies the boundary condition. A simple example of a second order boundary value problem is $y''(x) = y(x)$, $y(0) = 0$, $y(1) = 1$. The Neumann boundary condition is a type of boundary condition, named after Carl Neumann. When imposed on an ordinary or a partial differential equation, it specifies the value that the derivative of a solution is to take on the boundary of the domain. In Mathematics, a spherical coordinate system is a coordinate system for three dimensional space where the position of a point is specified by three numbers, the radial distance of the point from a fixed origin, its polar angle measured from a fixed zenith direction and the azimuth angle of its orthogonal projection on a reference plane that passes thru the origin and is orthogonal to the zenith measured from a fixed reference direction on that





plane. The concept of spherical coordinates can be extended to higher dimensional spaces and are then referred to as hyperspherical coordinates.

Result and Discussion:

The number of different spherical coordinate system following other conventions are used outside mathematics. In a geographical coordinate system position are measured in latitude, longitude and height or altitude. There are number of different celestial coordinate system based on different fundamental planes and with different terms for the various coordinates. The spherical coordinate systems used in mathematics normally use radians rather than degree and measure the azimuthal angle counter – clockwise rather than clockwise by H.W. Lord and Y. Shulman (3). The inclination angle is often replaced by the elevation angle measured from the reference plane. Elevation angle of zero is at the horigon. To define a spherical coordinate system, one must choose two orthogonal directions the zenith and the azimuth reference and an origin point in space. These choices determine a reference plane that contains the origin and is perpendicular to the zenith. The spherical coordinates of a point P are then defined as follows : The radius or radial distance is the Euclidean distance from the origin O to P. The inclination is the angle between the zenith direction and the line segment OP. The azimuth is the signed angle measured from the azimuth reference direction to the orthogonal projection of the line segment OP on the reference plane. Conventions : Several different conventions exist for representing the three coordinates and for the order in which they should be written. The use of (r, θ, Φ) to denote respectively, radial distance, inclination (or elevation) and azimuth is common practice in physics and is specified by ISO standard. The angles are typically measured in degrees ($^{\circ}$) or radians (rad), where $360^{\circ} = 2\pi$ rad. Degrees are most common in geography, astronomy and engineering where as radians are commonly used in mathematics and theoretical physics. The unit for radial distance is usually determined by the context. When the system is used for





physical three space, it is customary to use positive sign for azimuth angles that are measured in the counter – clockwise sense of the plane. This convention is used in particular for geographical coordinates where the “zenith” direction is north and positive azimuth (longitude) angles are measured eastwards from some prime meridian. Major Conventions : Coordinate Directions Right/Left Handed (r, θ_{inc} , Φ_{az} , right) (U,S,E) Right (r, Φ_{az} , right θ_{el}) (U,E,N) Right (r, θ_{el} , Φ_{az} , right) (U,N,E) Left Note : easting (E), northing (N), upwardness (U) Unique Coordinates : Any spherical coordinate triplet (r, θ , Φ) specifies a single point of three dimensional spaces. On the other hand, every point has infinitely many equivalent spherical coordinates. One can add or subtract any number of full turns to either angular measure without changing the angles themselves and therefore without changing the point. It is also convenient, in many contexts to allow negative radial distances, with the convention the (-r, θ , Φ) is equivalent to (r, $\theta + 180^\circ$, Φ) for any r, θ , Φ . Moreover, (r, $-\theta$, Φ) is equivalent to (r, θ , $\Phi + 180^\circ$). If it is necessary to define a unique set of spherical coordinates for each point one may restrict their ranges. A common choice is : $r \geq 0$ $0^\circ \leq \theta \leq 180^\circ$ $0^\circ \leq \Phi \leq 360^\circ$ However, the azimuth Φ is often restricted to the interval $(-180^\circ, +180^\circ]$, or $(-\pi, +\pi)$ in radians. Instead of $(^\circ, 360^\circ)$. This is the standard convention for geographic longitude. The range $(^\circ, 360^\circ)$ for inclination is equivalent to $[-90^\circ, +90^\circ]$ for elevation (latitude) used by Joseph (1994) (4). Coordinate System Conversions : As the spherical coordinate system is only one of many three-dimensional coordinate systems, there exist equations for converting coordinate between the spherical coordinate system and others. Cartesian Coordinate: The spherical coordinates (radius r, inclination θ , azimuth θ) of a point can be obtained from its cartesian coordinates (x, y, z) by the formulas. $r = \sqrt{(x^2 + y^2 + z^2)}$ $\theta = \arccos (z/r)$ $\Phi = \arctan (y/x)$ These formulae assume that the two systems have the same origin, that the spherical reference plane is the Cartesian xy plane, that θ is inclination from the Z direction and that the azimuth angles are measured from the Cartesian x axis (so that the y axis has $\Phi = +90^\circ$). If θ measures elevation





from the zenith they arc cos above becomes an arc sin by renal A.D and Schmidt B.G. (5). Existence and Uniqueness of Solution : For a large class of initial value problems, the existence and uniqueness of a solution can be illustrated through the use of a calculator. The Picard Lindelof theorem guarantees a unique solution on some interval containing to if f is continuous on a region containing to and y_0 and satisfies the Lipschitz condition on the variable y . The proof of this theorem by Burnett G.A (1) proceeds by reformulating the problem as an equivalent integral equation. The integral can be considered an operator which map one function into another, such that the solution is a fixed point of the operator. The Banach fixed point theorem is then invoked to show that there exists a unique fixed point, which is the solution of the initial value problem. The result may be found in Coddington & Levinson(2). An even more general result is the Caratheodory existence theorem, which proves existence for some discontinuous function f .

Exponential Smoothing : Exponential smoothing is a general method for removing noise from a data series, or producing a short term forecast of time series data for estimating initial value there are several methods, like we use these two formulas. $y_0' = (\alpha / (1-\alpha)) a_t + b_t y_0'' = (\alpha / (1-\alpha)) a_t + [2b]_t$

Examples : A simple example is to solve $y' = 0.85y$ and $y(0) = 19$. We are trying to find a formula for $y(t)$ that satisfies these two equation. Start by noting that $y' = dy/dt$, so $dy/dt=0.85y$ Now rearrange the equation so that y is on the left and t on the right. $dy/y=0.85 dt$ Now integrate both sides $\ln(|y|) = 0.85 t + B$ Eliminate the $\ln |y| = e^{B \cdot 0.85t}$ Let c be a new unknown constant $c = \pm e^B$ So, $y = c \cdot e^{0.85t}$ Now we need to find a value of C , use $y(0)=19$ as given so $19=c \cdot e^{0.85(0)} = c(1) = c$ $C = 19$ This gives the final solution of $y(t) = 19 e^{0.85t}$

In mathematical models and computer simulations, periodic boundary conditions (PBC) are a set of boundary conditions that are often used to simulate a large system by modeling a small part that is far from its edge.





Conclusion:

Periodic boundary condition (PBC) requires the unit cell to be a shape that will tile perfectly into a three dimensional crystal. Thus, a spherical or elliptical droplet cannot be used. A cube or rectangular prism is the most intuitive and common choice, but can be computationally expensive due to unnecessary amount of solvent molecules in the corners, distract from the central macromolecules. A common alternative that requires less volume is the truncated octahedron.

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Reference:

Burnett GA 1991 phys. Rev. D431143

Coddington and Levinson (1955) Partial differential equation.

H.W.Lord and Y. Shulmann, A generalized dynamic theory of thermoelasticity
J. Mech. Phys, solids 15 (1967) 299-309.

Joseph (1994), Principles of Algebraic geometry wiley interscience, ISBN
0471.50598

Rendall A.D. and Schmidt B.G. 1991 class Quantum Grav 8985.

