



BIANCHI TYPE – V ELECTROMAGNETIC STRING COSMOLOGICAL MODEL IN BIMETRIC RELATIVITY

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Abstract:

Bianchi Type V, Electromagnetic String Cosmological Model in Bimetric Theory of relativity is investigated. Here we have discuss some physical and kinematical properties of given model, further we have obtained vacuum model, which is free from singularity at cosmic time $t = 0$.

Keywords: Bianchi Type V Model, Cosmic String, Electromagnetic Field, Hubble parameter, deceleration parameter, Bimetric Relativity.

Introduction:

The study of Bianchi Type V, Cosmological model plays a vital role in the study of universe and it is keen interesting to note that, this model contain isotropic special case and also permit arbitrary small isotropic level as some point of time. The cosmic cloud string plays an important and significant role in the study of physical situation and at early stages of the formation of universe.

It is also assume that, the universe have under gone a series of phase transition as its temperature was lower down below some critical temperature. It is wellknown that symmetry of universe broken spontaneously which is due to topological stable defects such as domain walls, cosmic string, monopoles etc. among them cosmic string are the most interesting because they lead to the formation of galaxy.

Bianchi Type V universe is the generalization of FRW universe. Roy and Prasad (13) investigated Bianchi Type V model in general theory of relativity with the matter perfect fluid coupled with heat conduction and radiation. So many researchers studied this model such as Farnsworth (5), Collins(3), Maarthens and Nel (7), Wainwright et al (14), Beesham(2), Maharaj and Beesham(8), Pradhan et. at (9), Ayodogdu and Salti (1), Yadao A.K., Yadao V.K., Yadao L. (15).

In addition to this Quereshi A.A. & Deo S.D. (10), Deo S.D. and Ronge A.K. (4) also studied Bianchi Type V Cosmogocal model with various matter in Bimetric Theory of relativity. In continuation of this study, here we have discuss bianchi type V cosmogical model with the matter cosmic string coupled with electromagnetic field in Bimetric theory of relativity and discuss some physical and kinematical properties of model and further we observed that the resulting vacuum space time is free from singularity at cosmic time $t = 0$.

THE METRIC AND FIELD EQUATIONS:

We consider Bianchi Type V metric in the form

$$ds^2 = - dt^2 + A^2 dx^2 + e^{2x} B^2 (dy^2 + dz^2) \quad (1)$$

Where A, B and C are functions of t

We consider the background metric as

$$d\sigma^2 = - dt^2 + dx^2 + dy^2 + dz^2 \quad (2)$$

As we know that the field equations of Bimetric Relativity proposed by Rosen N (11, 12) are

$$K_{ij} = N_{ij} - N g_{ij} = -8\pi k \frac{1}{2} T_{ij} \quad (3)$$

$$\text{Where, } N_{ij} = \gamma^\beta [g^{hj} g_{hi} - \gamma^\alpha]_{i\beta} \quad (4)$$

$$\text{and } N = N = N^1_1 + N^2_2 + N^3_3 + N^4_4 \quad (5)$$

$$(6) \quad \text{where } g = \det(g_{ij}), \quad K = \frac{g}{\gamma} \quad \text{and } \gamma = \det(\gamma_{ij})$$

and vertical Bar (|) denotes the covariant differentiation with respect to γ_{ij}

and T_{ij} is energy momentum tensor for Cosmic strings coupled with electromagnetic field defined as.

$$T_{ij} = T_{ij}(\text{strings}) + T_{ij}(\text{e.m.f.})$$

$$\text{Here } T_{ij}(\text{strings}) = \rho v_i v_j - \lambda x_i x_j \quad (7)$$

Where v_i and x_i satisfy the condition

$$- v_i v^i = x_i x^i = 1,$$

Where ρ is the proper energy density of the cloud of string with particle attached to them, λ is string tension density, v^i is the four velocity of the particles and x^i is the unit space vector representing the direction of strings i.e. z-axis If the particle density of the configuration is denoted by ρ_p , then we have

$$\rho = \rho_p + \lambda \quad (8)$$

and $T_{ij}^{(emf)} = E_{ij}$ **defined as**

$$E_{ij} = -F_{ir} F^{ir} + \frac{1}{4} F_{ab} F^{ab} g_{ij} \tag{9}$$

Where E_{ij} is the electromagnetic energy tensor
 F_{ij} is the electromagnetic field tensor.

The magnetic field is taken along the x-direction, so that the only nonzero component of F_{ij} is $F_{23} = -F_{32}$

We have Maxwell's equation

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \tag{10}$$

gives rise to $F_{23} = -F_{32} = F$ (Constant) **(11)**

Using equations (1) to (11)

We have

$$\left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) - 2\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right) = 8\pi k \left(\frac{F^2}{e^{4xB^4}}\right) = 8\pi k w \tag{12}$$

$$\left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) = 8\pi k \left(\frac{F^2}{e^{4xB^4}}\right) = 8\pi k w \tag{13}$$

$$\left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) = 8\pi k \left(2\lambda + \frac{F^2}{e^{4xB^4}}\right) = 8\pi k [2\lambda + w] \tag{14}$$

$$\left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) + 2\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right) = 8\pi k \left(2\rho - \frac{F^2}{e^{4xB^4}}\right) = 8\pi k [2\rho - w] \tag{15}$$

Where $w =$

$$\frac{F^2}{e^{4xB^4}} \dot{A} = \frac{\partial A}{\partial t} \ddot{A} = \frac{\partial^2 A}{\partial t^2}$$

1. SOME PHYSICAL PROPERTIES :

The average scale factor for L.R.S. Bianchi Type V model is defined as

$$a = (AB^2)^{1/3},$$

Volume scale factor is given by

$$V = a^3 = AB^2$$

The generalized mean Hubble parameter H is given by

$$H = \frac{(H_x + H_y + H_z)}{3}$$

Where $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$, $H_z = \frac{\dot{B}}{B}$

are the directional Hubble parameter in the direction of x, y and z axes respectively.

The physical quantities expansion scalar θ , the average anisotropy parameter A_p and shear scalar σ^2 are defined as,

$$\theta = u^i_{;i}$$

$$A_p = \frac{1}{3} \sum_{i=1}^3 \left(\frac{\Delta H_i}{H}\right) = \frac{\dot{A}}{3A} + 2 \frac{\dot{B}}{B}$$

Where $\Delta H_i = H_i - H$ ($i = 1, 2, 3$)

$$\text{and } \sigma^2 = \frac{\sigma_{ij} \sigma^{ij}}{2} = \frac{1}{3} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2$$

2. SOLUTION OF FIELD EQUATIONS :

By using the equations (12) to (15)

We get

$$w + \lambda - \rho = 0 \tag{16}$$

According to Howking S.W. and Ellise G.F.R. (6)

For perfect fluid coupled with Maxwells field give rise to

$$\begin{aligned} \epsilon + p + 2w &\geq 0, \epsilon + p \geq 0 \\ \epsilon - p + 2w &\geq 0, \epsilon - p \geq 0 \end{aligned} \tag{17}$$

ϵ is energy density of the matter & p is the pressure of the fluid

Since $w \geq 0$ and it is sufficient to have

$$\epsilon + p \geq 0, \epsilon - p \geq 0 \tag{18}$$

Thus the equation no. (16)

$$w + \lambda - \rho = 0$$

is analogues of equation (17)

Using the equation (17) to (18)

We have, each terms of LHS of equation (16) is vanishes separately

$$\therefore \text{ie. } w = 0, \lambda = 0, \rho = 0$$

Thus in Binchi Type V universe cosmic cloud string as well as electromagnetic field does not exist, hence vaccum solution can be obtained

Thus the vaccum solution are

$$(19) \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) - 2\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right) = 0$$

$$(20) \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) = 0$$

$$(21) \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) + 2\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right) = 0$$

Solving equations (19) to (21)

We get

$$A = B = e^{mt + q}$$

Where m and q are constant of integration and absorbing the constant q in the differential and using the values of A and B in the equation no. (1) we obtain the vaccum model

$$ds^2 = -dt^2 + e^{2mt} [dx^2 + e^{2x} (dy^2 + dz^2)] \quad (22)$$

and it is interesting to note that the vacuum model (22) is free from singularity at cosmic time $t = 0$

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