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BIANCHI TYPE – V COSMOGICAL MODEL WITH MESSONIC FIELD AND DOMAIN WALL IN BIMETRIC RELATIVITY

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Abstract :
In this paper we have discussed spatially homogeneous and isotropic Bianchi type V Cosmogical Model with the matter massive messon and domain wall respectively. Further we have discuss Physical and Kinematical properties of the model.

Keywords :
Bianchi Type V model, Massive meson, Domain wall, Hubble parameter, deceleration parameter, Bimetric relativity.

1. Introduction :

In the past few decades their has been much more interest in the study of alternative theory of gravitation especially scalar tensor theory proposed by Brans & Diken (2), Nordvedt (8), Barber (1), Saez and Ballester (13), Lau and Prokhovnik (7) and bimetric theory of relativity proposed by Rosen N. (11) etc.

In the framework of general theory of relativity Rosen (1940) has proposed a bimetric theory of relativity which is based on two metrics

$$ds^2 = g_{ij}dx^i\, dx^j$$
and
$$d\sigma^2 = \gamma_{ij}dx^i\, dx^j$$

where ds is the interval between two neibouring events measured by means of a clock and measuring rod, the interval $d\sigma$ is an abstract or geometrical quantity not directly measurable. One can regard it as describing a geometry that would exist if no matter where present.

The fundamental metric tensor g_{ij} describe the gravitational potential when as a background flat metric γ_{ij} describe the inertial forces associated with the acceleration of the frame of reference. The metric tensor g_{ij} determine the Riemannian geometry of curve space time which placed the same role as given in Einstein theory of relativity and it interacts with matter, where as background metric γ_{ij} referred to the geometry of empty univers i.e. no matter gravition is there and it appears in the field equations i.e. one can regards γ_{ij} as flat space time having no physical significant. And it is important to note that bimetric theory is free from singularity which appears in general theory of relativity.

The spacially homogeneous and isotropic cosmological models play an important role in the description of universe at its early stages of evalution. Bianchi type I, II …….. IX cosmological models are very useful in cosmology spacially homogeneous model. Bianchi type V Models are very important because of to the fact that the space of costant negative curvature contains in it as a special case. So many authers have studied cosmological models in the context of general theory of relativity and bimetric theory of relativity respectively.

Bianchi type V cosmological codel has been studied by number of researcher such as Farnsworth (5), Shri Ram, M. Zeyauddin and C.P. Singh (14), R.K. Tiwari and Jaiswal (15), D.R.K. Reddy and N.V. Rao (10), in general theory of relativity and also A.A. Qureshi and S.D. Deo (9), S.D. Deo and A.K. Ronge (3), Deo & Rajurkar (4). Studied bianchi type V cosmological model with various matters in bimetric theory of relativity.

In continuation of this study, here we have discussed bianchi type V cosmological model with the matter massive messon and domain wall respectively in the bimetric theory of relativity and have discussed some physical and kinematical properties of this model.

2. THE METRIC AND FIELD EQUATIONS :

We consider Bianchi Type V cosmological model is of the form

$$ds^2 = -\,dt^2 + A^2\, dx^2 + e^{2x}\,(B^2\, dy^2 + C^2\, dz^2) \qquad (2.1)$$

Where A, B and C are function of t and consider the background metric for the equation (2.1) as

$$d\sigma^2 = -\,dt^2 + dx^2 + dy^2 + dz^2 \qquad (2.2)$$

and the field equations of Bimetric Relativity proposed by Rosen N (1940, 1973) are

$$K_i^j = N_i^j - \tfrac{1}{2} N g_i^j = -8\pi k T_i^j \quad (2.3)$$

Where,

$$N_i^j = \tfrac{1}{2} \gamma^{\alpha\beta} [g^{hj} g_{hi|\alpha}]_{|\beta} \quad (2.4)$$

and $N = N_\alpha^\alpha = N_1^1 + N_2^2 + N_3^3 + N_4^4$ (2.5)

$$K = \sqrt{\frac{g}{\gamma}} \quad (2.6)$$

where $g = \det(g_{ij})$

$\gamma = \det(\gamma_{ij})$

and vertical Bar (l) denotes the covariant differentiation with respect to γ_{ij}

and T_i^j is energy momentum tensor for the matter.

3. SOME PHYSICAL AND KINEMATICAL PROPERTIES :

The average scale factor **a**, the spatial volume **V** and the generalized Hubble's parameter **H** for the space time (2.1) are defined as

$$a = (ABC)^{1/3} \quad (3.1)$$

$$V = a^3 = ABC \quad (3.2)$$

$$H = \frac{1}{3}(H_x + H_y + H_z) \quad (3.3)$$

Where, $H_x = \frac{(A)_4}{A}$, $H_y = \frac{(B)_4}{B}$, $H_z = \frac{(C)_4}{C}$ are the directional Hubble's parameters in direction of x,y,z respectively. The suffix 4 denotes differentiation with respect to t

From equation (3.1) – (3.3), we have the important relation

$$H = \frac{(a)_4}{a} = \frac{1}{3}\left(\frac{(A)_4}{A} + \frac{(B)_4}{B} + \frac{(C)_4}{C}\right) \quad (3.4)$$

The kinematical quantities such as scalar expansion (θ), the shear scalar (σ^2) and the anisotropy parameter (A_m) are defined as

$$\theta = u_{;\mu}^{\mu} \quad (3.5)$$

$$\sigma^2 = \frac{1}{2} \sigma_{\mu\nu} \sigma^{\mu\nu} \quad (3.6)$$

$$A_m = \frac{1}{3}\sum_{\mu=1}^{3}\left(\frac{H\mu - H}{H}\right)^2 \quad (3.7)$$

Where $u^\mu = (0, 0, 0, 1)$ is the matter 4 – velocity vector and

$$\sigma_{\mu\nu} = \frac{1}{2}(u_{\mu;\alpha} P_\nu^\alpha + u_{\nu;\alpha} P_\mu^\alpha) - \frac{1}{3}\theta P_{\mu\nu} \quad (3.8)$$

Here the projection tensor $P_{\mu\nu}$ has the form

$$P_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu \quad (3.9)$$

These dynamical scalars, in Bianchi type V have the form

$$\theta = \frac{(A)_4}{A} + \frac{(B)_4}{B} + \frac{(C)_4}{C} \quad (3.10)$$

$$2\sigma^2 = \frac{(A_4)^2}{A} + \frac{(B_4)^2}{B} + \frac{(C_4)^2}{C} - \frac{\theta^2}{3} \quad (3.11)$$

and the deceleration parameter q in cosmology is given by

$$q = \frac{-a(a)_{44}}{(a_4)^2} \quad (3.12)$$

4. SOLUTION OF FIELD EQUATIONS :

Here we will consider the matter massive meson and domain wall respectively then

Case I : Massive Messon

The energy momentum tensor for the matter massive messon is defined as

$$T_i^j = V_{,i} V^{,j} - \frac{1}{2} g_i^j \left(V_{,k} V^{,k} - m^2 v^2 \right) \quad (4.1)$$

Together with $\sigma = g_i^j V_{;I}^{\ j} + m^2 v,$

$$V_4 V^4 = 1$$

Where m is the mass parameter and σ is the source density of the messon field. Here after suffix (,) and semicolon (;) after a field variable represent ordinary and covariant differentiation with respect to x^i and g_i^j respectively.

$$T_1^1 = -\frac{1}{2} [V_4^2 - m^2 v^2] \quad (4.2)$$

$$T_2^2 = -\frac{1}{2} [V_4^2 - m^2 v^2] \quad (4.3)$$

$$T_3^3 = -\frac{1}{2} [V_4^2 - m^2 v^2] \quad (4.4)$$

$$T_4^4 = -\frac{1}{2} [V_4^2 + m^2 v^2] \quad (4.5)$$

using the equations (2.1) to (4.5) we have

$$- \left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = 8\pi k \; (V_4^2 - m^2 v^2) \quad (4.6)$$

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = 8\pi k \; (V_4^2 - m^2 v^2) \quad (4.7)$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 = 8\pi k \; (V_4^2 - m^2 v^2) \quad (4.8)$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = 8\pi k \; (V_4^2 - m^2 v^2) \quad (4.9)$$

Solving equations (4.6) – (4.9)

We get

$$V_4^2 - m^2 v^2 = 0 \quad (4.10)$$

Thus in Bianchi type V model massive messon does not exit.

i.e. vacuum model can be obtained

From equation (4.6) – (4.10)

We get

$$\left(\frac{A_4}{A}\right)_4 = \left(\frac{B_4}{B}\right)_4 = \left(\frac{C_4}{C}\right)_4 = d \quad (4.11)$$

i.e. $\left(\frac{A_4}{A}\right)_4 = d$, where d is constant

using the equation (4.11) we get

$$A = B = C = e^{\lambda t^2 + k_1 t + k_2} \quad (4.12)$$

Absorbing the constant K_2 in differential then metric (2.1) becomes

$$ds^2 = -dt^2 + e^{2(\lambda t^2 + k_1 t)}[dx^2 + e^{2x}(dy^2 + dz^2)] \quad (4.13)$$

Case II : Domain Wall

Here we will consider the energy momentum tensor for Domain wall

$$T_i^j = T_i^j{}_{wall} = \rho\,(g_i^j + w_i w^j) + p\,w_i w^j \quad (4.14)$$

Where **ρ** is energy density of the wall, **p** is the pressure of the direction normal to the plane of wall and w_i is the unit space like vector in the same direction with $w_i w^j = -1$

In particular $w_1 w^1 = -1$

Then we have

$$T_1^1 = p,\ T_2^2 = T_3^3 = T_4^4 = \rho \quad (4.15)$$

Now equation **(2.3)** becomes

$$-\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = 16\pi k p \quad (4.16)$$

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = -16\pi k \rho \quad (4.17)$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 = -16\pi k \rho \quad (4.18)$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = -16\pi k \rho \quad (4.19)$$

Solving equation **(4.16) – (4.19)**

We get

$$O = p - \rho \quad (4.20)$$

According to Hawking S.W and Ellise G.F.R. (6) for perfect fluid,

We have

$$\left.\begin{array}{l} \epsilon + p \geq 0 \\ \epsilon - p \geq 0 \end{array}\right\} \quad (4.21)$$

Where **ϵ** is energy density of the matter and **p** is the pressure of the fluid

Thus equation **(4.20)** is the analog of equation **(4.21)**

Thus each term of L.H.S. of equation **(4.20)** is vanishes separately i.e.

$$p = o = \rho \quad (4.22)$$

Thus in the Bianchi type V universe domain wall does not survive.

Hence vacuum solution can be obtained.

We get same result as equation **(4.13)**

$$ds^2 = -dt^2 + e^{2(\lambda t^2 + k_1 t)}\,dx^2 + e^{2x}(dy^2 + dz^2)]$$

and it is interesting to note that the model **(4.13)** is free from singularity at $t = 0$

Conclusion :

In Bimetric theory of relativity when we study Bianchi Type V model with matter massive messon and domain wall respectively. We observe that the matters massive meson and domain wall does not exist respectively in this theory, thus we obtain a vacuum solution which is free from singularity at t = 0.

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