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STABILITY OF BIANCHI TYPE-IX COSMOLOGICAL MODEL IN BRANS-DICKE THEORY OF GRAVITATION

A. S. Nimkar[1], J. S. Wath[2]
[1]Department of Mathematics, Shri. Dr. R. G. Rathod Arts & Science College, Murtizapur, Dist. Akola (M.S.) India.
[2]Department of Applied Mathematics, P.R. Pote (Patil) College of Engineering and Management, Amravati.
anilnimkar@gmail.com, jotsnawath@gmail.com

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ABSTRACT:
In this paper, we have investigated stability of Bianchi Type-IX cosmological model in the presence of energy momentum tensor for matter and the holographic dark energy in the framework of scalar tensor theories of gravitation proposed by Brans-Dicke[1]. To obtain the exact solution we have used variation law for Hubble parameter. Also, we discuss the physical and kinematic properties of the model.

Key words: - Bianchi Type-IX space-time, Brans-Dicke, Matter and the holographic dark energy

INTRODUCTION:

In the present paper, we have used Bianchi Type-IX cosmological model. Several researchers are using Bianchi Type-IX space time as a result of this space time model permits not solely expansion however conjointly rotation and shear.

Recently, there has been a lot of interest in alternative theories of gravitation, especially, the Brans-Dicke[1] theory of gravity. Brans-Dicke theory introduces a scalar field φ which has the dimension of the inverse of gravitational constant and which interacts uniformly with all forms of matter. Brans-Dicke theory is a powerful scalar-tensor theory due to its huge cosmological implications.

Brans-Dicke field equations for combined scalar and tensor fields are given by

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{'k}\right) - \phi^{-1}\left(\phi_{i;j} - g_{ij}\square\phi\right) \quad (1)$$

and

$$\square\phi = \phi^{'k}_{;k} = 8\pi\phi^{-1}(3+2\omega)^{-1}(T_{ij} + \overline{T}_{ij}) \quad (2)$$

The energy conservation equation is,

$$T^{ij}_{;j} + \overline{T}^{ij}_{;j} = 0 \quad (3)$$

In the recent years, the holographic principle plays an important role in solving the dark energy problem, by using this principle Guberina et al.[2], Setare[3], M. Kiran et al[4], Farajollahi et al[5], Mete et al[6], S.Wang et al[7], Jing-Fei Zang et al[8], are some of the authors who have studied various aspects of holographic dark energy and dark energy

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models using various theories. Also, D. D. Pawar et al[9] examined Kantowski-sach model in brance-dicke theory of gravitation. S. D. Katore et al[10], Parth Shah et al[11], Geovanny A. Rave Franco et al[12] have analyzed stability of cosmological models. Bali et al[13], Tyagi et al[14], Bagora[15] has investigated Bianchi Type IX cosmological model. In this paper, we obtained cosmological model using a linearly varying deceleration parameter by Akarsu and Dereli[16].

The purpose of the present work is to obtain stability of Bianchi Type-IX cosmological model in the presence of energy momentum tensor for matter and the holographic dark energy. Our paper is organized as follows. In section 2, Metric and field Equations. Section 3, Solutions of field Equations. The last section contains some conclusion.

Metric and field Equations

Consider the Bianchi type-IX metric in the form

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + (b^2\sin^2 y + a^2\cos^2 y)\,dz^2 - 2a^2\cos y\,dx\,dz \qquad (4)$$

Where a and b are functions of time t alone. The energy momentum tensor for matter and the holographic dark energy are defined as

$$T_{ij} = \rho_m u_i u_j \quad \text{and}$$

$$\bar{T}_{ij} = (\rho_\lambda + p_\lambda)u_i u_j + g_{ij}\, p_\lambda \qquad (5)$$

where ρ_m, ρ_λ are the energy densities of matter and the holographic dark energy, p_λ is the pressure of the holographic dark energy and u_i is the four velocity vectors of the distribution respectively.

Also the scalar field satisfies the following equation

$$\Box\phi = \phi^{,k}_{;k} = 8\pi\phi^{-1}(3+2\omega)^{-1}(T+\bar{T})$$

From Eq. (5), we have

$$T_1^1 = T_2^2 = T_3^3 = p_\lambda$$

and $$T_4^4 = -(\rho_m + \rho_\lambda) \qquad (6)$$

Using the equations (1) - (3) and (5) - (6), the field equations of metric (4) are

$$\frac{2b_{44}}{b} + \frac{b_4^{\,2}}{b^2} + \frac{1}{b^2} - \frac{3a^2}{4b^4} + \frac{\omega}{2}\frac{\phi_4^{\,2}}{\phi^2} + \frac{2b_4}{b}\frac{\phi_4}{\phi} + \frac{\phi_{44}}{\phi} = -8\pi\phi^{-1}p_\lambda \qquad (7)$$

$$\frac{a_{44}}{a} + \frac{b_{44}}{b} + \frac{a_4b_4}{ab} + \frac{a^2}{4b^4} + \frac{\omega}{2}\frac{\phi_4^{\,2}}{\phi^2} + \left(\frac{a_4}{a} + \frac{b_4}{b}\right)\frac{\phi_4}{\phi} + \frac{\phi_{44}}{\phi} = -8\pi\phi^{-1}p_\lambda \qquad (8)$$

$$\frac{2a_4b_4}{ab} + \frac{b_4^{\,2}}{b^2} - \frac{a^2}{4b^4} + \frac{1}{b^2} - \frac{\omega}{2}\frac{\phi_4^{\,2}}{\phi^2} + \left(\frac{a_4}{a} + \frac{2b_4}{b}\right)\frac{\phi_4}{\phi} + \frac{\phi_{44}}{\phi} = -8\pi\phi^{-1}(\rho_m + \rho_\lambda) \qquad (9)$$

$$\left(\frac{a_4}{a} + \frac{b_4}{b}\right)\phi_4 + \phi_{44} = \frac{8\pi\phi^{-1}}{(3+2\omega)}(3p_\lambda - \rho_m - \rho_\lambda) \qquad (10)$$

By the law of conservation we have,

$$(\rho_m)_4 + \rho_m\left(\frac{a_4}{a} + \frac{2b_4}{b}\right) + (\rho_\lambda)_4 + (1+\omega_\lambda)\rho_\lambda\left(\frac{a_4}{a} + \frac{2b_4}{b}\right) = 0 \qquad (11)$$

Where the subscript '4' after a, b, ϕ and ρ denote partial differentiation with respect to t.

3. Solutions of field Equations

The above system of the equations (7) to (11) are five equations and seven unknown $a, b, \phi, \omega_\lambda, p_\lambda, \rho_m$ & ρ_λ. In order to solve this undermined system two additional constraints are required.

Hence to find a determinate solution we use the following conditions which physically corresponds the vanishing of trace of both matter and dark energy tensors. This is analogous to the disordered radiation condition of general relativity.
(M. Kiran[4])

$$T + \bar{T} = \rho_m + \rho_\lambda - 3p_\lambda = 0$$

So, from above equation we get,

$$3p_\lambda - \rho_m - \rho_\lambda = 0 \tag{12}$$

Here we assume minimally interacting matter and holographic dark energy components, both the components conserve separately, hence can be written as

$$(\rho_m)_4 + \rho_m\left(\frac{a_4}{a} + \frac{2b_4}{b}\right) = 0 \tag{13}$$

$$(\rho_\lambda)_4 + (1+\omega_\lambda)\rho_\lambda\left(\frac{a_4}{a} + \frac{2b_4}{b}\right) = 0 \tag{14}$$

Using barotropic equation of state

$$p_\lambda = \rho_\lambda \omega_\lambda$$

where ω_λ is the EoS parameter for holographic dark energy through which we investigate disparate epoch of universe.

For Bianchi type IX cosmological model, the average scale factor R and the spatial volume V are given by

$$R(t) = \left(ab^2\right)^{\frac{1}{3}} \tag{15}$$

$$V = R^3 = ab^2 \tag{16}$$

The mean generalized Hubble's parameter H for this model is given by

$$H = \frac{1}{3}\left(H_x + H_y + H_z\right) = \frac{1}{3}\left(\frac{a_4}{a} + 2\frac{b_4}{b}\right) \tag{17}$$

where H_x, H_y and H_z are the directional Hubble parameters along *x, y* and *z* axes respectively defined by

$$H_x = \frac{a_4}{a}, \qquad H_y = H_Z = \frac{b_4}{b} \tag{18}$$

The average anisotropic parameter A_m for the universe is defined as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2$$

where $\Delta H_i = H_i - H$ (19)

The anisotropic parameter can be used to inspect whether the universe expands anisotropically or isotropically. The universe expands anisotropically for nonzero value of anisotropic parameter and that of it expands isotropically if $A_m = 0$.

The expansion scalar θ and shear scalar σ^2 are given by

$$\theta = 3H = \left(\frac{a_4}{a} + 2\frac{b_4}{b}\right)$$

(20)

$$\sigma^2 = \frac{3}{2}A_m H^2 = \frac{1}{3}\left(\frac{a_4}{a} - \frac{b_4}{b}\right)^2$$

(21)

Also, we use the condition that the shear scalar σ is directly proportional to expansion scalar θ.

$$\sigma \propto \theta \quad \Rightarrow \sigma = k_1\theta$$

(22)

where k_1 is a proportionality constant. Using equations (20) and (21),

Equation (22) becomes,

$$\frac{1}{\sqrt{3}}\left(\frac{a_4}{a} - \frac{b_4}{b}\right) = k_1\left(\frac{a_4}{a} + \frac{2b_4}{b}\right)$$

(23)

On integrating equation (23) we get,

$$a = b^{\left(\frac{2\sqrt{3}\,k_1+1}{1-\sqrt{3}\,k_1}\right)}$$

(24)

Now, we consider a generalized linearly varying deceleration parameter (Akarsu and Dereli[17])

$$q = \frac{-RR_{44}}{R_4{}^2} = -k_2 t + n - 1$$

(25)

Where $k_2 \geq 0$ and $n \geq 0$ are constants and $k_2 = 0$ reduces to the law of Berman (1983) which yields models with constant deceleration parameter.

Solving (25), one obtained following different form of solution for the scale factor

$$R = k_3 e^{k_4 t}$$ for $k_2 = 0 \;\&\; n = 0$

$$R = k_5(nt + k_6)^{\frac{1}{n}}$$ for $k_2 = 0 \;\&\; n > 0$

Where $k_3,\, k_4,\, k_5,\, k_6$ are constants of integration.

3.1 Case I - For $k_2 = 0 \;\&\; n = 0$, average scale factor is

$$R = k_3 e^{k_4 t}$$

(26)

From equations (15), (24) and (26) we get,

$$a = k_7 e^{k_1(2\sqrt{3}\,k_1+1)t}$$

(27)

$$b = k_8 e^{k_1(1-\sqrt{3}\,k_1)t}$$

(28)

With the help of equations (10), (12), (27) and (28) we get,

$$\phi = \frac{k_{10}}{e^{k_1(2+\sqrt{3}\,k_1)t}}$$

(29)

Equations (16) and (26) gives spatial volume,

$$V = k_{11} e^{3k_1 t}$$

(30)

The mean generalized Hubble's parameter H for this model defined by (17) takes the form

$$H = k_1$$

(31)

The expansion scalar θ and shear scalar σ^2 are defined by (20) and (21) takes the form

$$\theta = 3k_1$$

(32)

$$\sigma^2 = 9k_1^{\,4}$$

(33)

The average anisotropy parameter is,

$$A_m = 6k_1^{\,2}$$

(34)

By using equations (27) and (28), equation (4) takes the form

$$ds^2 = -dt^2 + \left(k_7 e^{k_1(2\sqrt{3}k_1+1)t}\right)^2 dx^2 + \left(k_8 e^{k_1(1-\sqrt{3}k_1)t}\right)^2 dy +$$

$$\left[\left(k_8 e^{k_1(1-\sqrt{3}k_1)t}\right)^2 \sin^2 y + \left(k_7 e^{k_1(2\sqrt{3}k_1+1)t}\right)^2 \cos^2 y\right] dz^2 - 2\left(k_7 e^{k_1(2\sqrt{3}k_1+1)t}\right)^2 \cos y z\, dx dz$$

(35)

Using equation (13), the energy density of matter is given by

$$\rho_m = \frac{k_{12}}{e^{3k_1 t}}$$

(36)

The Holographic dark energy pressure is given by

$$P_\lambda = \frac{k_{13}}{8\pi e^{k_1(2+\sqrt{3}k_1)t}} \left\{S_1 + S_2 e^{-2k_1(1-\sqrt{3}k_1)t} - S_3 e^{6k_1 t} + S_4\right\}$$

(37)

The Holographic dark energy density is given by

$$\rho_\lambda = \frac{3k_{13}}{8\pi e^{k_1(2+\sqrt{3}k_1)t}} \left\{S_1 + S_2 e^{-2k_1(1-\sqrt{3}k_1)t} - S_3 e^{6k_1 t} + S_4 - S_5 e^{k_1(\sqrt{3}k_1-1)t}\right\}$$

(38)

The EoS parameter is given by

$$\omega_\lambda = \frac{\left\{S_1 + S_2 e^{-2k_1(1-\sqrt{3}k_1)t} - S_3 e^{6k_1 t} + S_4\right\}}{3\left\{S_1 + S_2 e^{-2k_1(1-\sqrt{3}k_1)t} - S_3 e^{6k_1 t} + S_4 - S_5 e^{k_1(\sqrt{3}k-1_1)t}\right\}}$$

(39)

where

$$S_1 = 3k_1^{\,2}\left(1-\sqrt{3}k_1\right)^2, \qquad S_2 = \frac{1}{k_8^{\,2}},$$

$$S_3 = \frac{3}{4}\frac{k_7^{\,2}}{k_8^{\,4}},$$

$$S_4 = \left(\frac{\omega}{2} - \frac{2\left(1-\sqrt{3}k_1\right)}{\left(2+\sqrt{3}k_1\right)} + 1\right) k_1^{\,2}\left(2+\sqrt{3}k_1\right)^2,$$

$$S_5 = \frac{8\pi k_{12}}{3k_{13}}$$

3.1.1 Kinematics properties of the model-

The decomposition of time-like tidal tensor is

$$u_{\alpha;b} = -\left[k_1 k_7^{\,2}(2\sqrt{3}k_1+1)e^{2k_1(2\sqrt{3}k_1+1)t}(1+\cos^2 y) + k_1 k_8^{\,2}(1-\sqrt{3}k_1)e^{2k_1(1-\sqrt{3}k_1)t}(1+\sin^2 y)\right]$$

And

Vorticity $\omega_{11} = \omega_{22} = \omega_{33} = \omega_{44} = 0$

Vorticity of model along *x, y, z* and *t*- axes is zero. So, the obtained model is non-rotating. Whereas vorticity is nonzero, model is rotating.

3.1.2 Stability solution and physical significance of the model-

We discuss the stability of the model by observing the ratio of sound speed given by $\frac{dp_\lambda}{d\rho_\lambda} = c_s^{\,2}$, when the ratio $\frac{dp_\lambda}{d\rho_\lambda}$ is positive i.e. $c_s^{\,2} > 0$, we have a stable model

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whereas when the ratio $\frac{dp_\lambda}{d\rho_\lambda}$ is negative i.e. $c_s^{\,2} < 0$, we have an unstable model.

Also, in this model Hubble parameter, expansion scalar, shear scalar, and the average anisotropy parameter are constant throughout the universe except spatial volume is a function of time. The mean generalized Hubble parameter is constant throughout the evolution of the universe so $\frac{dH}{dt} = 0$. This shows that greater the value of Hubble parameter, faster the rate of expansion of universe.

3.2 Case II -

For $k_2 = 0 \;\&\; n > 0$, average scale factor is

$$R = k_5 (nt + k_6)^{\frac{1}{n}}$$

(40)

From equations (15), (24) and (40) we get,

$$a = k_{15} (nt + k_6)^{\frac{2\sqrt{3}k_1 + 1}{n}}$$

(41)

$$b = k_{16} (nt + k_6)^{\frac{1 - \sqrt{3}\,k_1}{n}}$$

(42)

With the help of equations (10), (12), (41) and (42) we get,

$$\phi = k_{17} (nt + k_6)^{\frac{n + 2 - \sqrt{3}\,k_1}{n}}$$

(43)

Equations (16) and (40) gives spatial volume,

$$V = k_{18} (nt + k_9)^{\frac{3}{n}}$$

(44) The mean generalized Hubble's parameter H for this model defined by (17) takes the form

$$H = \frac{1}{(nt + k_9)}$$

(45)

The expansion scalar θ and shear scalar σ^2 are defined by (20) and (21) takes the form

$$\theta = \frac{3}{(nt + k_9)}$$

(46)

$$\sigma^2 = \frac{9k_1^{\,2}}{(nt + k_9)^2}$$

(47)

The average anisotropy parameter is,

$$A_m = 6k_1^{\,2}$$

(48)

By using equations (41) and (42), equation (4) takes the form

$$ds^2 = -dt^2 + \left(k_{15} (nt + k_6)^{\frac{2\sqrt{3}k_1 + 1}{n}} \right)^2 dx^2 + \left(k_{16} (nt + k_6)^{\frac{1 - \sqrt{3}\,k_1}{n}} \right)^2 dy^2$$
$$+ \left\{ \left(k_{16} (nt + k_6)^{\frac{1 - \sqrt{3}\,k_1}{n}} \right)^2 \sin^2 y + \left(k_{15} (nt + k_6)^{\frac{2\sqrt{3}k_1 + 1}{n}} \right)^2 \cos^2 y \right\} dz^2 - 2\left(k_{15} (nt + k_6)^{\frac{2\sqrt{3}k_1 + 1}{n}} \right)^2 \cos yz \, dx dz$$

(49)

Using equation (13), the energy density of matter is given by

$$\rho_m = \frac{k_{19}}{(nt + k_6)^{\frac{3}{n}}}$$

(50)

The Holographic dark energy pressure is given by

$$P_\lambda = \frac{k_{17}(nt+k_6)^{\frac{n+2-\sqrt{3}k_1}{n}}}{8\pi}\left\{L_1(nt+k_6)^{-2} + L_2(nt+k_6)^{\frac{2\sqrt{3}k_1-2}{n}} - L_3(nt+k_6)^{\frac{8\sqrt{3}k_1-2}{n}} + L_4\right\}$$

(51)

The Holographic dark energy density is given by

$$\rho_\lambda = \frac{3k_{17}(nt+k_6)^{\frac{n+2-\sqrt{3}k_1}{n}}}{8\pi}\left\{L_1(nt+k_6)^{-2} + L_2(nt+k_6)^{\frac{2\sqrt{3}k_1-2}{n}} - L_3(nt+k_6)^{\frac{8\sqrt{3}k_1-2}{n}} + L_4 + L_5(nt+k_6)^{\frac{\sqrt{3}k_1-5-n}{n}}\right\}$$

(52)

The EoS parameter is given by

$$\omega_\lambda = \frac{\left\{L_1(nt+k_6)^{-2} + L_2(nt+k_6)^{\frac{2\sqrt{3}k_1-2}{n}} - L_3(nt+k_6)^{\frac{8\sqrt{3}k_1-2}{n}} + L_4\right\}}{3\left\{L_1(nt+k_6)^{-2} + L_2(nt+k_6)^{\frac{2\sqrt{3}k_1-2}{n}} - L_3(nt+k_6)^{\frac{8\sqrt{3}k_1-2}{n}} + L_4 + L_5(nt+k_6)^{\frac{\sqrt{3}k_1-5-n}{n}}\right\}}$$

(53)

where

$$L_1 = 12k_1^2 - \sqrt{3}k_1 n - 10\sqrt{3}k_1 + 7 + \frac{\omega}{2}\left(n+2-\sqrt{3}k_1\right)^2$$

$$L_2 = \frac{1}{k_{16}^2}, \qquad L_3 = \frac{3}{4}\frac{k_{15}^2}{k_{16}^4},$$

$$L_4 = \left(2-2\sqrt{3}k_1\right)\left(n+2-\sqrt{3}k_1\right),$$

$$L_5 = \frac{8\pi k_{19}}{3k_{17}}.$$

3.2.1 Kinematics properties of the model-

The decomposition of time-like tidal tensor is

$$u_{a;b} = -\left[k_{15}^2(2\sqrt{3}k_1+1)(nt+k_6)^{\frac{2(2\sqrt{3}k_1+1)-n}{n}}(1+\cos^2 y) + k_{16}^2(1-\sqrt{3}k_1)(nt+k_6)^{\frac{2(1-\sqrt{3}k_1)-n}{n}}(1+\sin^2 y)\right]$$

And

Vorticity $\omega_{11} = \omega_{22} = \omega_{33} = \omega_{44} = 0$

Vorticity of model along *x, y, z* and *t*- axes is zero. So, the obtained model is non-rotating. Whereas vorticity is nonzero, model is rotating.

This shows that the given cosmological model is stable throughout the evolution of the universe for $k_2 = 0$ & $n > 0$.

Also, in this model spatial volume, Hubble parameter, expansion scalar and shear scalar are the function of cosmic time throughout the universe except the average anisotropy parameter is constant.

3.2.2 Stability solution and physical significance of the model-

We discuss the stability of the model by observing the ratio of sound speed given by $\frac{dp_\lambda}{d\rho_\lambda} = c_s^2$, when the ratio $\frac{dp_\lambda}{d\rho_\lambda}$ is positive i.e. $c_s^2 > 0$, we have a stable model whereas when the ratio $\frac{dp_\lambda}{d\rho_\lambda}$ is negative i.e. $c_s^2 < 0$, we have an unstable model.

The behavior of Spatial Volume and Hubble Parameter are represented in Fig (5) and Fig. (6), as time increases Spatial Volume increases. But Hubble Parameter decreases and finally vanish at $t \to \infty$.

It is also interesting to note that, all the above parameters vanish when cosmic time is infinite whereas when cosmic time is zero all the above parameters are finite.

CONCLUSION

In these paper, we have investigated stability of Bianchi Type-IX cosmological model in BD scalar tensor theories of gravitation. We found the stability of cosmological model in case I and case II with the help energy momentum tensor for matter and the holographic dark energy. We have obtained the exact solution of Bianchi Type-IX cosmological model.

It is also interesting to note that, in both the cases (Both the model) the average anisotropy parameter A_m is same and constant. In both the models EoS parameter ω_λ is a function of time through which we investigate disparate epoch of universe.

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In this model,

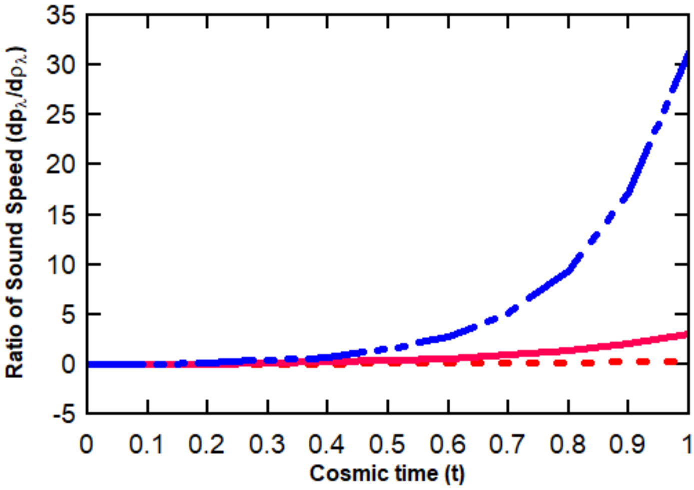

Fig. (1) Plot of ratio of sound speed $\frac{dp_\lambda}{d\rho_\lambda} = {c_s}^2$ versus cosmic time (t)

From Fig (1), it is observed that initially ratio of sound speed ${c_s}^2$ is constant. But after sometime when time increases ${c_s}^2$ also increases and goes to infinity. This shows that the given cosmological model is stable at initial stage for $k_2 = 0 \;\&\; n = 0$.

In this model,

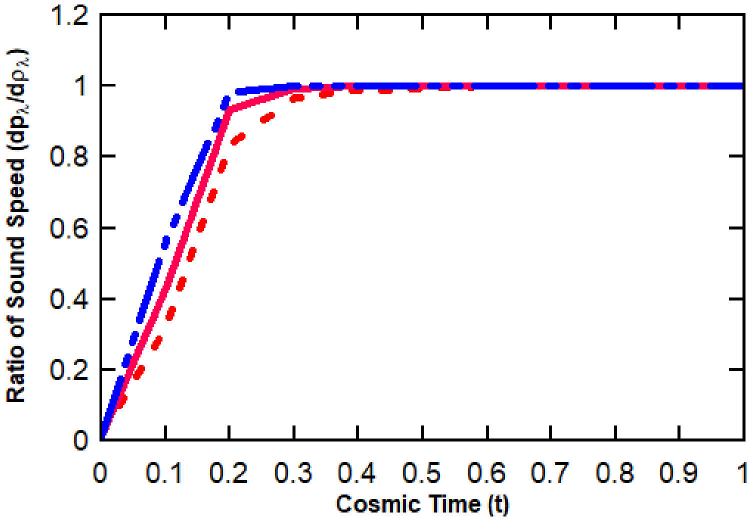

Fig. (2) Plot of ratio of sound speed $\frac{dp_\lambda}{d\rho_\lambda} = {c_s}^2$ versus cosmic time (t)

From Fig (2), it is observed that the ratio of sound speed ${c_s}^2$ increases as time increases afterwards times increases but ratio of sound speed ${c_s}^2$ is constant. This shows that the given cosmological model is stable throughout the evolution of the universe for $k_2 = 0 \;\&\; n > 0$.

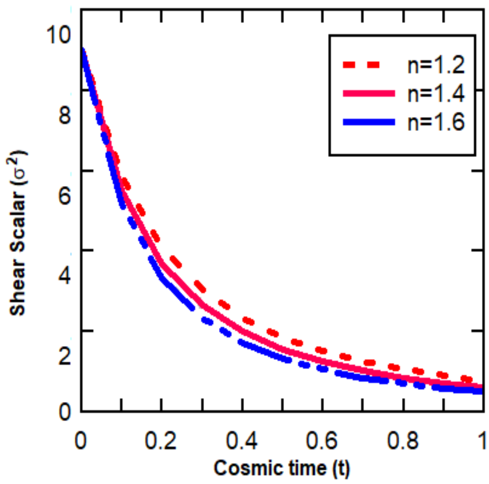

Fig. 3 Plot of Shear Scalar Vs. Cosmic time (t)

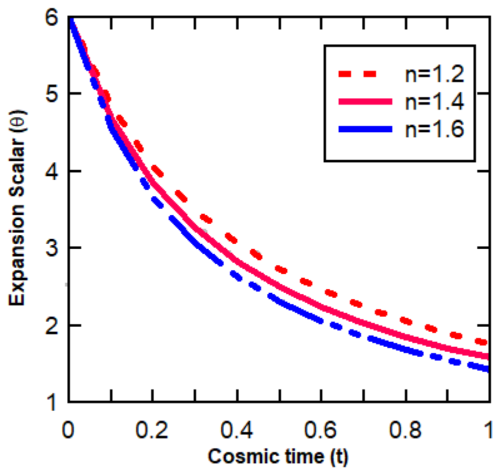

Fig. 4 Plot of Expansion Scalar Vs. Cosmic time (t)

The behavior of Shear scalar and expansion scalar are represented in Fig (3) and Fig. (4), as time increases shear scalar and expansion scalar decreases and finally they vanishes at $t \to \infty$

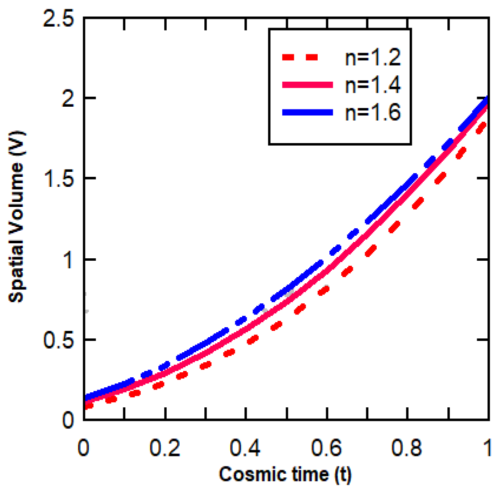

Fig. 5 Plot of Spatial Volume Vs. Cosmic time (t)

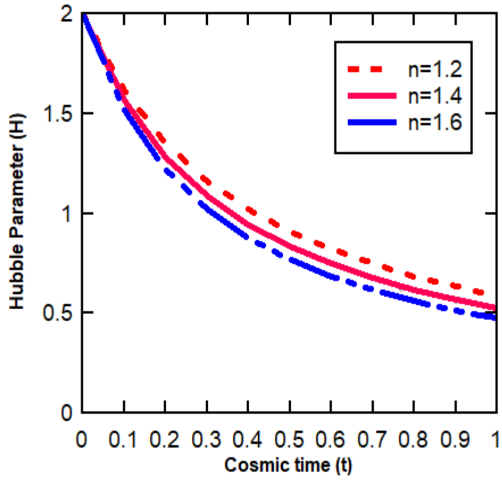

Fig. 6 Plot of Hubble Parameter Vs Cosmic time (t)

The behavior of Spatial Volume and Hubble Parameter are represented in Fig (5) and Fig. (6), as time increases Spatial Volume increases. But Hubble Parameter decreases and finally vanish at $t \to \infty$.

It is also interesting to note that, all the above parameters vanish when cosmic time is infinite whereas when cosmic time is zero all the above parameters are finite.