



A COMPARATIVE STUDY OF COSMOLOGICAL MODELS IN SEAZ BALLESTER THEORY OF GRAVITATION

A. S. NIMKAR & S. R. HADOLE

* Shri. Dr. R. G. Rathod Arts And Science College, Murtizapur, Dist. Akola, (M.S.) India
anilnimkar@gmail.com, sangita.hadole2000@gmail.com

Communicated: 30.07.20

Revision :11.08.20 & 30.8.2020

Accepted: 17.09.2020

Published: 30.09.2020

ABSTRACT:

In this paper, we have investigated Bianchi type-II cosmological models in Saez -Ballester theory under the influence of cosmic string, perfect fluid and thick domain wall, wet dark fluid and macroscopic body. Exact cosmological models in the theory are obtained with the help of relation between metric coefficient and equation of state. We also discussed various physical and dynamical properties of the models. The variation of different cosmological parameters are shown graphically for specific values of the parameters of the models. The main aim of the paper is to compare the obtained results of cosmic string, Perfect fluid thick domain wall ,wet dark fluid, and macroscopic body within the framework of Saez -Ballester theory.

Key words: - Bianchi type-II, Saez Ballester theory, cosmic string, Perfect fluid, thick domain wall, wet dark fluid, and macroscopic body .

INTRODUCTION:

Scalar-Tensor Theories of Gravitation are considered to be the most natural generalization of general relativity. Saez Ballester [1] constructed a scalar tensor theory of gravitation in which the metric is coupled with dimensionless scalar field in a simple manner .This coupling gives a satisfactory description of the weak field.

Saez Ballester theory of gravitation commits the field equations as

$$G_{ij} - \omega \phi^n \left(\phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k} \right) = -8\pi T_{ij} \quad (1)$$

$$2\phi^n \phi_{,i}^i + n\phi^{n-1} \phi_{,k} \phi^{,k} = 0 \quad (2)$$

Where $G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}$ is the Einstein tensor,

R_{ij} is the Ricci tensor, R is the scalar curvature,

n is arbitrary constant, ω is dimensionless

coupling constant and T_{ij} is energy –momentum tensor. Here comma and semicolon denote partial and covariant differentiation respectively.

Saez[2], Sing & Agarwal[3], Shri Ram &Singh [4], Shri Ram Tiwari[5] , Reddy& Rao[6] ,Rao, Sireesha & Suneetha [7] and Sharma &Singh [8] are some of the authers who have studied various aspect of Saez Ballester theory.

The purpose of the present work is to obtain Bianchi type-II Cosmological model in presence of cosmic string, perfect fluid, thick domain walls, wet dark fluid and macroscopic body. Our paper is organized as follows. In section 2, we derive the Metric and field Equations in Seaz-Ballester theory of gravitation. Section 3, Solutions of cosmic string as a source, Section 4, Solutions of perfect fluid as a source. Section 5, Solutions of thick domain walls. Section 6 contains solutions of wet dark fluid. Section 7, Solutions of macroscopic body. Section 8 is mainly concerned

with the physical and Kinematical properties of the model and section9,graphical representation of cosmic string, perfect fluid, thick domain walls and wet dark fluid and macroscopic body.Section 10 contains the observational parameters of all the models. The last section contains comparisons and conclusion.

2. Metric and Field Equations:

We consider the Bianchi type II space time in the form,

$$ds^2 = dt^2 - A^2(dx - zdy)^2 - B^2dy^2 - C^2dz^2 \quad (3)$$

Where A, B,C are function of t only.

The field equation in Seaz Ballester theory are given by

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} - \frac{3A^2}{4B^2C^2} - \frac{\omega}{2}\phi^n\phi_4^2 = 8\pi T_1^1 \quad (4)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} + \frac{A^2}{4B^2C^2} - \frac{\omega}{2}\phi^n\phi_4^2 = 8\pi T_2^2 \quad (5)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \frac{A^2}{4B^2C^2} - \frac{\omega}{2}\phi^n\phi_4^2 = 8\pi T_3^3 \quad (6)$$

$$\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} - \frac{A^2}{4B^2C^2} - \frac{\omega}{2}\phi^n\phi_4^2 = 8\pi T_4^4 \quad (7)$$

$$\phi_{44} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) \phi_4 + \frac{n\phi_4^2}{2\phi} = 0 \quad (8)$$

Where the suffix '4' on the following unknown represent ordinary differentiation w.r.to 't' only.

3.Solution of Cosmic string:

The investigation for cosmic string is one of the interesting problem of morden astronomy, cosmology and particle physics.The concept of

cosmic string is hypothetical one dimensional object at cosmological scale.The concept of string theory was established to express events of the early stage of the formation of the universe.The general relativistic treatment of strings was initiated by Vinenkin[9] and Letelier[10] and the gravitational effects of cosmic strings have been extensively discussed by,Khadekar[11], Kandalkar[12],Adhavet. al.[13],Pawaret. al. [14],Vidyasagar *et.al.* [15].

The energy momentum tensor for string cloud is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \quad (9)$$

where ρ is the rest energy density of the cloud of strings with massive particles attached to them, $\rho = \rho_p + \lambda, \rho_p$ being the rest energy of the particles attached to the strings and λ the tension density of the system of strings. u_i describes the cloud four velocity and x_i represents the .direction of strings.

From equation (9)

$$T_1^1 = \lambda, T_2^2 = T_3^3 = 0, T_4^4 = \rho \quad (10)$$

The equations (4) to (8) is a system of six independent equations with seven unknown $A, B, C, \phi, \rho, \lambda$ and ω In order to get deterministic solution we use the condition for Reddy String

$$\rho + \lambda = 0 \quad (11)$$

Also the power law is given by

$$A = B^n \quad \text{and} \quad C = B^m \quad (12)$$

Using (4) to (8), we get,

$$BB_{44} + k_1 B_4^2 = k_2 B^{2(n-m)} \quad (13)$$

Solving equation (13) we get,

$$A = (k_4 t + k_5)^{\frac{n}{m-n+1}} \quad (14)$$

$$B = (k_4 t + k_5)^{\frac{1}{m-n+1}} \tag{15}$$

$$C = (k_4 t + k_5)^{\frac{m}{m-n+1}} \tag{16}$$

With assist the equation (14),(15) and (16) equation (3) of the form

$$ds^2 = dt^2 - (k_4 t + k_5)^{\frac{2n}{m-n+1}} (dx - zdy)^2 - (k_4 t + k_5)^{\frac{2}{m-n+1}} dy^2 (k_4 t + k_5)^{\frac{2m}{m-n+1}} dz^2 \tag{17}$$

From equation (8),we get

$$\phi = \frac{k_8}{(k_4 t + k_5)^{\frac{4n}{(n+2)(m-n+1)}}} \tag{18}$$

$$\lambda = \frac{k_4^2}{8\pi(k_4 t + k_5)^2} \left(\frac{n + nm + m}{(m - n + 1)^2} - \frac{3}{4k_4^2} - \frac{\omega k_7^2}{2k_4^2(k_4 t + k_5)^{\frac{4n}{m-n+1}}} \right) \tag{19}$$

$$\rho = \frac{k_4^2}{8\pi(k_4 t + k_5)^2} \left(\frac{n + nm + m}{(m - n + 1)^2} - \frac{1}{4k_4^2} + \frac{\omega k_7^2}{2k_4^2(k_4 t + k_5)^{\frac{4n}{m-n+1}}} \right) \tag{20}$$

Spatial Volume $V = (k_4 t + k_5)^{\frac{m+n+1}{m-n+1}} \tag{21}$

Scalar Expansion $\theta = \frac{k_6}{3(k_4 t + k_5)} \tag{22}$

Hubble Parameter $H = \frac{k_6}{(k_4 t + k_5)} \tag{23}$

Shear Scalar $\sigma^2 = \frac{k_6^2}{54(k_4 t + k_5)^2} \tag{24}$

Deceleration parameter $q = -1 + \frac{(m - n + 1)^2}{m + n + 1} \tag{25}$

Graphical representation of spatial Volume, Shear scalar, Hubble Parameter and Expansion Scalar with time of cosmic string are in Fig-1.

Solution of Perfect fluid:

We have the perfect fluid energy momentum tensor as -

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{26}$$

$$u^i u^j = 1 \tag{27}$$

Where u^i is the four velocity vector of the fluid and P and ρ are the proper pressure and energy density respectively.

From (26) and (27), the components of T_i^j in commoving coordinate are,

$$T_1^1 = -p, \quad T_2^2 = T_3^3 = -p, \quad T_4^4 = \rho \tag{28}$$

Condition of stiff fluid $p - \rho = 0 \tag{29}$

Solving equations (4) to (8) with assist of equations (28) and (29), we get metric coefficients, cosmological model, physical properties and graphical illustration of the model. But all above results are identical the result obtained from cosmic string. Also the value of proper pressure and energy density are

$$p = \frac{-k_4^2}{8\pi(k_4 t + k_5)^2} \left(\frac{n + nm + m}{(m - n + 1)^2} - \frac{3}{4k_4^2} - \frac{\omega k_7^2}{2k_4^2(k_4 t + k_5)^{\frac{4n}{m-n+1}}} \right) \tag{30}$$

$$\rho = \frac{k_4^2}{8\pi(k_4 t + k_5)^2} \left(\frac{n + nm + m}{(m - n + 1)^2} - \frac{1}{4k_4^2} + \frac{\omega k_7^2}{2k_4^2(k_4 t + k_5)^{\frac{4n}{m-n+1}}} \right) \tag{31}$$

5. Solution and Model of Thick Domain walls:

In this section we discuss the thick domain walls in the Bianchi type-II space time given by (3). A thick domain wall may be considered as a solution like solution of the scalar field equations coupled with gravity. There are two methods of analyzing thick domain walls. One method is to solve gravitational field equations with an energy-momentum tensor describing a scalar field ψ with self interactions contained in a potential $v(\psi)$ given by

$$\psi_i \psi_j - g_{ij} \left[\frac{1}{2} \psi_i \psi_j - v(\psi) \right] \quad (32)$$

Second approach is to assume the energy momentum tensor in the form

$$T_{ij} = \rho (g_{ij} + \omega_i \omega_j) + p \omega_i \omega_j \quad (33)$$

$$\omega_i \omega^j = -1 \quad (34)$$

Where ρ is energy density of the walls, p is pressure in direction normal to the plane of the wall and ω_i is a unit space-like vector in the same direction.

Katore *et al.* [16], Chirde *et al.* [17], Bhoyare *et al.* [18] are some of the authors who have investigated several aspects of domain wall.

Here we use the second approach to study the thick domain walls in scale covariant theory of gravitation.

From (33) with the help of (34), we get

$$T_1^1 = -p, \quad T_2^2 = T_3^3 = T_4^4 = \rho \quad (35)$$

$$\text{Using radiation condition } p - 3\rho = 0 \quad (36)$$

Solving equations (4) to (8) with the help of equation (35) and (36).

We obtain metric coefficients, cosmological model, physical properties and graphical illustration of the model. But all above results are identical the result obtain from cosmic string and perfect fluid. Also the value of proper pressure and energy density are,

$$p = \frac{-k_4^2}{8\pi(k_4 t + k_5)^2} \left(\frac{n + nm + m}{(m - n + 1)^2} - \frac{3}{4k_4^2} - \frac{\omega k_7^2}{2k_4^2(k_4 t + k_5)^{\frac{4n}{m-n+1}}} \right) \quad (37)$$

$$\rho = \frac{k_4^2}{8\pi(k_4 t + k_5)^2} \left(\frac{n + nm + m}{(m - n + 1)^2} - \frac{1}{4k_4^2} + \frac{\omega k_7^2}{2k_4^2(k_4 t + k_5)^{\frac{4n}{m-n+1}}} \right) \quad (38)$$

6. Solution and model of Wet dark fluid:

The nature of the dark energy component of the universe as one of the hidden mysteries of cosmology. The wet dark fluid (WDF) as a model for dark energy. This model is in the essence of the generalized Chaplygin gas Gorini *et al.* [19], where an energetic equation of state is offered with properties relevant for the dark energy problem. Here the motivation stems from an empirical equation of state proposed by Tait [20] and Hayward [21] to treat water and aqueous solution.

The equation of state for WDF is very simple.

$$P_{WDF} = \gamma(\rho_{WDF} - \rho_*)$$

The pressure γ and ρ_* are taken to be positive and we restrict ourselves to $0 \leq \gamma \leq 1$

To find the WDF energy density, we use the energy conservation equation

$$\rho_{WDF} = \frac{\gamma}{1 + \gamma} \rho_* + \frac{c}{v(1 + v)}$$

Where c is the constant of integration and v is the volume expansion WDF naturally includes two components, a piece that behaves as a cosmological constant as well as a standard fluid with an equation of state $P = \gamma\rho$. We can show that if we take $c > 0$, this fluid will not violate the strong energy condition $p + \rho \geq 0$.

$$p_{WDF} + \rho_{WDF} = (1 + \gamma)\rho_{WDF} - \gamma\rho_* \\ = (1 + \gamma)\frac{c}{v^{(1+\gamma)}} \geq 0$$

The wet dark fluid has been used as dark energy in the homogeneous, isotropic FRW case by Holman *et. al.* [22]. T. Singh *et. al.* [23] studied in Bianchi type I universe with wet dark fluid. Adhavet. *al.*[24] have been studied in detailed for Einstein-Rosen universe with wet dark fluid. Also Chaubey[25], Mishra *et. al.*[26], Samanta[27], Nimkaret. *al.*[28], Chirde *et. al.*[29], Deo *et. al.*[30], Angit *et. al.*[31] are some of the researchers who have investigated various aspects of wet dark fluid.

The energy momentum tensor of wet dark fluid is given by

$$T_{ij} = (\rho_{WDF} + p_{WDF})u_i u_j - p_{WDF}g_{ij} \quad (39)$$

Where u^i is the flow vector satisfying

$$g_{ij}u^i u^j = 1 \quad (40)$$

By using comoving system of coordinates, we get

$$T_1^1 = T_2^2 = T_3^3 = -p_{WDF}, \quad T_4^4 = \rho_{WDF} \quad (41)$$

Use condition of stiff fluid $p - \rho = 0$ (42)

Solving equations (4) to (8) with assist of equation (41) and (42), we obtain metric coefficient,

cosmological model, physical properties and graphical illustration of the model. But all above results are indential the results obtain from cosmic string, perfect fluid and thick domain walls also. The Value of proper pressure and energy density are

$$p_{WDF} = \frac{-k_4^2}{8\pi(k_4 t + k_5)^2} \left(\frac{n + nm + m}{(m - n + 1)^2} - \frac{3}{4k_4^2} - \frac{\omega k_7^2}{2k_4^2(k_4 t + k_5)^{\frac{4n}{m-n+1}}} \right) \quad (43)$$

$$\rho_{WDF} = \frac{k_4^2}{8\pi(k_4 t + k_5)^2} \left(\frac{n + nm + m}{(m - n + 1)^2} - \frac{1}{4k_4^2} + \frac{\omega k_7^2}{2k_4^2(k_4 t + k_5)^{\frac{4n}{m-n+1}}} \right) \quad (44)$$

7. Solution and model of macroscopic body:

The energy momentum tensor of macroscopic body (Landue L.D and Lifshitz E.M) [32] is given by,

$$T^{ij} = (\mathcal{E} + p)u^i u^j - p g^{ij} \quad (45)$$

Here p is pressure, \mathcal{E} is energy density and u_i represents the four velocity vectors of the distribution respectively.

$$T_1^1 = T_2^2 = T_3^3 = -p, \quad T_4^4 = \mathcal{E} \quad (46)$$

The equation (5)-(8) contain seven unknown $A, B, C, \phi, \rho, p, \omega$. To find the deterministic solution use power relation $A = B^n, C = B^m$ and proper volume v and average scale factor $a(t)$ for Bianchi type II is

$$a(t) = (ABC)^{\frac{1}{3}} \quad (47)$$

$$v = [a(t)]^3 \quad (48)$$

The physical quantities of cosmological model are expansion scalar θ , the mean anisotropy parameter Am and shear scalar σ are defined as

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \quad (49)$$

$$H = \frac{1}{3}\theta \quad (50)$$

$$Am = \frac{1}{3} \sum_{i=1}^3 \left(\frac{\Delta H_i}{H} \right)^2 \quad (51)$$

$$\sigma^2 = \frac{3}{2} Am H^2 \quad (52)$$

The variation of Hubble's parameter suggested by Berman [34] that yields constant deceleration parameter of the model is given by

$$q = \frac{aa_{44}}{a_4^2} \quad (53)$$

Solving (47) and (53) we get

$$A = (C_3t + C_4)^{\frac{3n}{(q+1)(m+n+1)}} \quad (54)$$

$$B = (C_3t + C_4)^{\frac{3}{(q+1)(m+n+1)}} \quad (55)$$

$$C = (C_3t + C_4)^{\frac{3m}{(q+1)(m+n+1)}} \quad (56)$$

From equation (54),(55) and (56) cosmological model of equation (3) can be written as

$$ds^2 = dt^2 - (C_3t + C_4)^{\frac{6n}{(q+1)(m+n+1)}} (dx - zdy)^2 - (C_3t + C_4)^{\frac{6}{(q+1)(m+n+1)}} dz^2 - (C_3t + C_4)^{\frac{6m}{(q+1)(m+n+1)}} dt^2 \quad (57)$$

(57)

Scalar field, pressure and energy density is given by,

$$\phi = C_6 (C_3t + C_4)^{\frac{2(q-2)}{(q+1)(n+2)}} \quad (58)$$

Where
$$C_6 = \left[\frac{(q+1)2C_5}{(q-2)(n+2)} \right]^{\frac{2}{n+2}} \quad (59)$$

$$P = \frac{-3C_3^2}{8\pi(C_3t + C_4)^2} \left[\frac{C_7}{(q+1)^2(m+n+1)^2} - \frac{1}{4C_3^2(C_3t + C_4)^{\frac{6(m-n+1)}{(q+1)(m+n+1)-2}} - \frac{\omega C_3^2}{6C_3^2(C_3t + C_4)^{\frac{6}{(q+1)(m+n+1)-2}} \right] \quad (60)$$

(60)

$$\varepsilon = \frac{C_3^2}{8\pi(C_3t + C_4)^2} \left[\frac{9(m+n+mn)}{(q+1)^2(m+n+1)^2} - \frac{1}{4C_3^2(C_3t + C_4)^{\frac{6(m-n+1)}{(q+1)(m+n+1)-2}} + \frac{\omega C_3^2}{2C_3^2(C_3t + C_4)^{\frac{6}{(q+1)(m+n+1)-2}} \right] \quad (61)$$

(61)

Properties of model:

Sapatial volume
$$V = (C_3t + C_4)^{\frac{3}{q+1}} \quad (62)$$

Hubble parameter
$$H = \frac{C_3}{(q+1)(C_3t + C_4)} \quad (63)$$

Expansion scalar
$$\theta = \frac{3C_3}{(q+1)(C_3t + C_4)} \quad (64)$$

Average scale factor
$$a(t) = (C_3t + C_4)^{\frac{1}{q+1}} \quad (65)$$

Shear scalar

$$\sigma^2 = \frac{3C_3^2 (m^2 + n^2 - 1 - mn - m - n)}{(m+n+1)^2 (q+1)(C_3t + C_4)^2} \quad (66)$$

Graphical representation of spatial Volume, Shear scalar, Hubble Parameter and Expansion Scalar with time of macroscopic body are as Fig-2

Also, the expression for the energy density W , energy flow vector S and stress tensor $\sigma_{\alpha\beta}$ are

$$W = \left(\frac{1 + \frac{v^2}{3c^2}}{1 - \frac{v^2}{c^2}} \right) \frac{C_3^2}{8\pi(C_3t + C_4)^2} \left[\frac{9(m+n+mm)}{(q+1)^2(m+n+1)^2} - \frac{1}{4C_3^2(C_3t + C_4)^{\frac{6(m-n+1)}{(q+1)(m+n+1)-2}}} + \frac{\alpha C_3^2}{2C_3^2(C_3t + C_4)^{\frac{6}{(q+1)(m+n+1)-2}} \right] \tag{67}$$

$$S = \left(\frac{4v}{3 \left(1 - \frac{v^2}{c^2} \right)} \right) \frac{C_3^2}{8\pi(C_3t + C_4)^2} \left[\frac{9(m+n+mm)}{(q+1)^2(m+n+1)^2} - \frac{1}{4C_3^2(C_3t + C_4)^{\frac{6(m-n+1)}{(q+1)(m+n+1)-2}}} + \frac{\alpha C_3^2}{2C_3^2(C_3t + C_4)^{\frac{6}{(q+1)(m+n+1)-2}} \right] \tag{68}$$

$$\sigma_{\alpha\beta} = \left(\frac{4v_\alpha v_\beta}{c^2 \left(1 - \frac{v^2}{c^2} \right)} \right) + \delta_{\alpha\beta} \frac{-3C_3^2}{8\pi(C_3t + C_4)^2} \left[\frac{C_7}{(q+1)^2(m+n+1)^2} - \frac{1}{4C_3^2(C_3t + C_4)^{\frac{6(m-n+1)}{(q+1)(m+n+1)-2}}} - \frac{\alpha C_3^2}{6C_3^2(C_3t + C_4)^{\frac{6}{(q+1)(m+n+1)-2}} \right] \tag{69}$$

If the velocity of the microscopic motion is small compared with the velocity of the light, then we have approximately $S = (p + \epsilon)v$. Since $\frac{S}{c^2}$ is

the momentum density and $\frac{(p + \epsilon)}{c^2}$ plays the role the mass density of the body. From the expression (46), we get

$$T_i^j = \epsilon - 3p \tag{70}$$

But,
$$T_i^j = \sum_a m_a c^2 \sqrt{1 - \frac{v_a^2}{c^2}} \delta(r - r_0) \tag{71}$$

Compare the relation (70) with the general formula (71) which we saw was valid for an arbitrary system. Since we are at present considering a microscopic body, the expression (71) must be averaged over all the value of r in unit volume. We obtained the result

$$\epsilon - 3p = \sum_a m_a c^2 \sqrt{1 - \frac{v_a^2}{c^2}} \tag{72}$$

Here the summation extends over all particles in the unit volume

The right side of this equation tends to zero in the ultra relativistic limit, so in this the equation of

state of matter is
$$p = \frac{\epsilon}{3}$$

8. Properties of all four models:

Average Scale Factor $a(t) = (k_4 t + k_5)^{\frac{m+n+1}{3(m-n+1)}}$
$$J = \frac{2(m-2n+1)(5m-7n+5)}{27(m+n+1)^2} \tag{73}$$

Deceleration Parameter $q = -1 + \frac{(m-n+1)^2}{m+n+1}$
$$J = \frac{2(m-2n+1)(5m-7n+5)(-8m+10n-8)}{81(m+n+1)^3} \tag{74}$$

Jark Parameter $J = \frac{2(m-2n+1)(5m-7n+5)}{27(m+n+1)^2} \tag{75}$

Snap Parameter $J = \frac{2(m-2n+1)(5m-7n+5)(-8m+10n-8)}{81(m+n+1)^3} \tag{76}$

Lark parameter $J = \frac{2(m-2n+1)(5m-7n+5)(8m-10n+8)(11m-13n+11)}{213(m+n+1)^4} \tag{77}$

Properties of Microscopic body model:

Average Scale Factor $a(t) = (C_3 t + C_4)^{\frac{1}{(q+1)}}$
$$J = \frac{2(m-2n+1)(5m-7n+5)(8m-10n+8)(11m-13n+11)}{213(m+n+1)^4} \tag{78}$$

Jark parameter $J = (2q+1)q \tag{79}$

Observational parameter of Microscopic body Model:

From equation (78),(82) and (83) gives

Look back time:

$$t_L = \frac{C_3}{(q+1)H_0} \left[1 - \frac{1}{(1+z)^{\frac{1}{q+1}}} \right] \tag{86}$$

Snap parameter $S = -q(2q+1)(3q+2)$ (80)

Lark parameter $L = q(2q+1)(3q+2)(4q+3)$ (81)

9. Graphical Illustration:

Graphical illustration of spatial Volume, Shear scalar, Hubble Parameter and Expansion Scalar with time of cosmic string , perfect fluid ,wet dark fluid and domain walls and microscopic body are shown in fig 3

Observational parameters of all Four models:

Look-back time redshift:Look -back time is defined as the difference between present age of universe t_0 and the age of the universe t when a particular light ray at redshift z was emitted. It depends on the dynamics of the universe.

$$t_L = t_0 - t \tag{82}$$

Where t_0 is present age of universe and z denotes redshift of light well measured quantity of a far distant object such as galaxies. The redshift of light is emitted due to expansion of universe. For given redshift z ,the average scale factor of the universe $a(t)$ is related to the present scale factor of the universe $a_0(t)$ by

$$1+z = \frac{a_0(t)}{a(t)} \tag{83}$$

From equations (73), (82) and (83) give

$$t_L = \frac{k_6}{H_0} \left[1 - \frac{1}{(1+z)^{\frac{3(m-n+1)}{(m+n+1)}}} \right] \tag{84}$$

Luminosity Distance redshift: The Luminosity distance of light source is given

$$H_0 d_L = \frac{3k_6(m-n+1)}{(2m-4n+2)} \left[1 - (1+z)^{\frac{-(2m-4n+2)}{(m+n+1)}} \right] (1+z)$$

CONCLUSION

In brief, Energy momentum tensor and space-time associated with them have cosmological interest due to their important applications in structure formation of the universe. Also, it is well known that scalar fields have considerable effects in the early stages of revolutionary universe. Here we have presented Bianchi type II cosmological models in Saez Ballester theory of gravitation proposed by Saez[1] with cosmic string, Perfect fluid, thick domain walls, wet dark fluid and macroscopic body. The models in four energy momentum tensor are similar and it is observed that the macroscopic body could exist in the early epoch of the universe by using law of variation of Hubble’s parameter which yield constant deceleration parameter and average scale factor. The behavior of physical parameter of Bianchi type II macroscopic body cosmological model is similar to Rao et al [7].Also discussed some physical and kinematical properties and graphical illustration of cosmic string, Perfect fluid, thick domain walls, wet dark fluid and macroscopic body. All these models studied here will be useful for a better understanding of Saez Ballester cosmology and structure formation of the universe.

It may be observed that at initial moment, when $t=0$, the spatial volume will be zero while energy density and pressure diverge. When t tends to zero, then the expansion scalar , shear scalar and Hubble’s parameters tends to infinity. For large

value of t , we observe that expansion scalar, shear scalar and Hubble parameters become zero at late time

As we know the motivation behind the scalar tensor theories is to explain the accelerated expansion of universe.

REFERENCES:

- Saez ,D., Ballester, V.J.: Phys Lett.A **113**, 467 (1986)
- Saez, D.: Simple Coupling with Cosmological implications (preprint) (1985)
- Singh ,T.,Agarwal, A. K.: Astrophys Space Sci. **182**,289 (1991)
- Ram, S., Singh,J.K.:Astrophys Space Sci. **234**,325 (1995)
- Ram,S., Tiwari, S.K.: Astrophys Space Sci.**259**,91 (1998)
- Reddy, D.R.K., Rao, N.V.: Astrophys. Space Sci. **277**, 461 (2001)
- Rao,V.U.M.,SireeshaK.V.S.and Suneetha,P.:TheAfricanReviewof Phys.467(2012)7:0054
- Sharma,N.K.,Singh,J.K.:Int J Theor.Phys (2014) DOI 10.1007/s10773-14-2264-9
- Vilenkin, A.: Phys. Rev. D 23, 852 (1981)
- Letelier, P.S.: Phys. Rev. D 28, 2414 (1983)
- KhadekarG.S,TadeS.D.:Astrophysics and Space Sci.310 (1),47-51 (2007)
- KandalkarS.P ,KhadeP.P.,Gawande S.P. Bulg.J.Phys.38,pp 145-154 (2011)
- Adhav K.S., Mete V.G,Thakare R.S.,Pund A.K.:Int.J.Theor phys,50,1 pp 164-170 (2011)
- PawarD.D,Solanke Y.S.,Bayaskar S.N.: Prespace time Journal 5 (2).(2014)
- Vidyasagar T,Naidu R.L, Vijaya R.B.,ReddyD.R.K.:The European physics Journal plus,129,2 pp 1-7 (2014)
- KatoreS.D.,Hatkar S.P ,BaxiR.J.:Chinese Journal of physics ,Vol.54,issue 4, PP563-573 (2016)
- ChirdeV.R ,ShekhS.H.:Journal of Astrophysics and Astronomy.37 (2) ,15 (2016)
- BhoyarS.R.,Chirde V.R, ShekhS.H.:Astrophysics ,vol.60.No.2 (2017)
- GoriniV.,Kamenshchik,A. Moschella,U and Pasquier,V.:Xir:gr-qc/0403062(2004)
- Tait P.G.:The Voyage of HMS Challenger (1988)
- Hayward A T J.: Brit.J.Appl.phys.18,965 (1967)
- Holmani Singh T,Chaubey R .: Pramana Journal of physics,71,no.3 (2008)
- Singh,T. and Chaubey,R.:Pramana Journal of physics,71 No.3 (2008)
- Adhav Jain P,SahooP.K.,MishraB.:Int .J.Theor.phys,51,2 pp 2546-2551 (2012)
- ChaubeyR.:International Journal of Astronomy and Astrophysics,vol.1 No.2,(2011)
- Mishra B.,SahooP.K.:Astrophysics space sci.DOI10.1007/s10509-013-1652-6
- SamantaG.S.,JaiswalS,BiswalS.K.:The European physical Journal plus 129,no 48 (2014)
- NimkarA.S.,Pund A.M.:IOSRJournal of Mathematics,11 ,4,pp 47-50 (2015)
- ChirdeV.R.,KadamV.P.:International Journal for Innovative Research in Sciences &Technology ,vol.3.issue 05(2016)
- Deo S.D.,PunwatkarG.S.,PatilU.M.:International Journal of Mathematical Archive,vol.7.No.3(2016)
- AngitS.,Raushan Rakesh, ChaubeyR.:International Journal of Geometric methods inModern physics,vol.16.No.08 (2019)
- Landue L.D. and Lifshitz E.M.: The classical Theory of Fields Fourth Revised English Edition,Pergamon press.
- Berman, M.S.:Nuovo Cimento B.74 ,182 (1983)

Fig .1

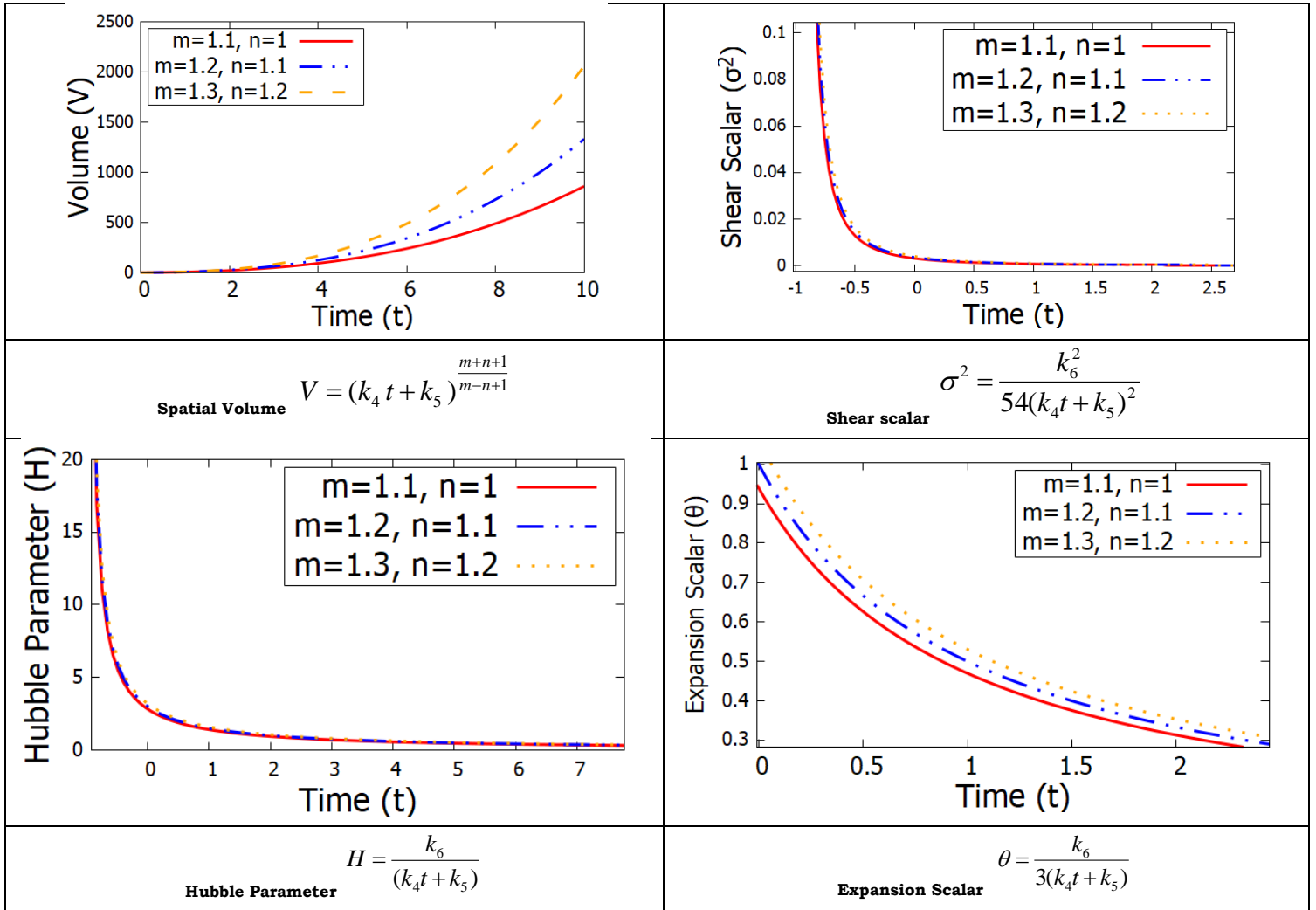


Figure 1 shows that volume increases with time. Volume $V \rightarrow \infty$ as $t \rightarrow \infty$. Hubble parameter decreases with increasing time. The expansion scalar and shear scalar are decreasing with increase in time and both vanishes as $t \rightarrow \infty$ for distinct value of m and n in presence of cosmic string Also , plot of Average Scale Factor verses time -

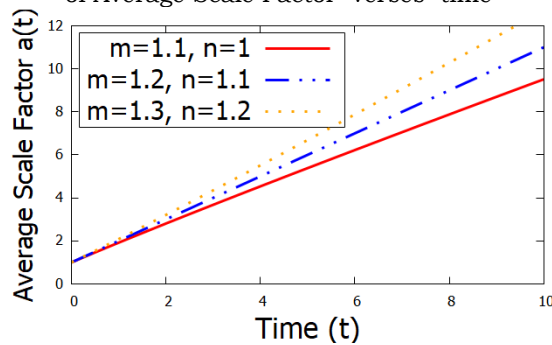


Fig. 2: Plot of Average Scale Factor vs. Time

Where, $k_4 = k_5 = 1$

Fig.2

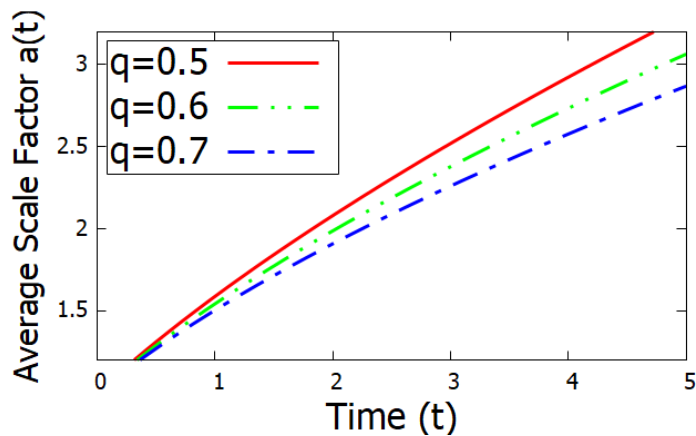
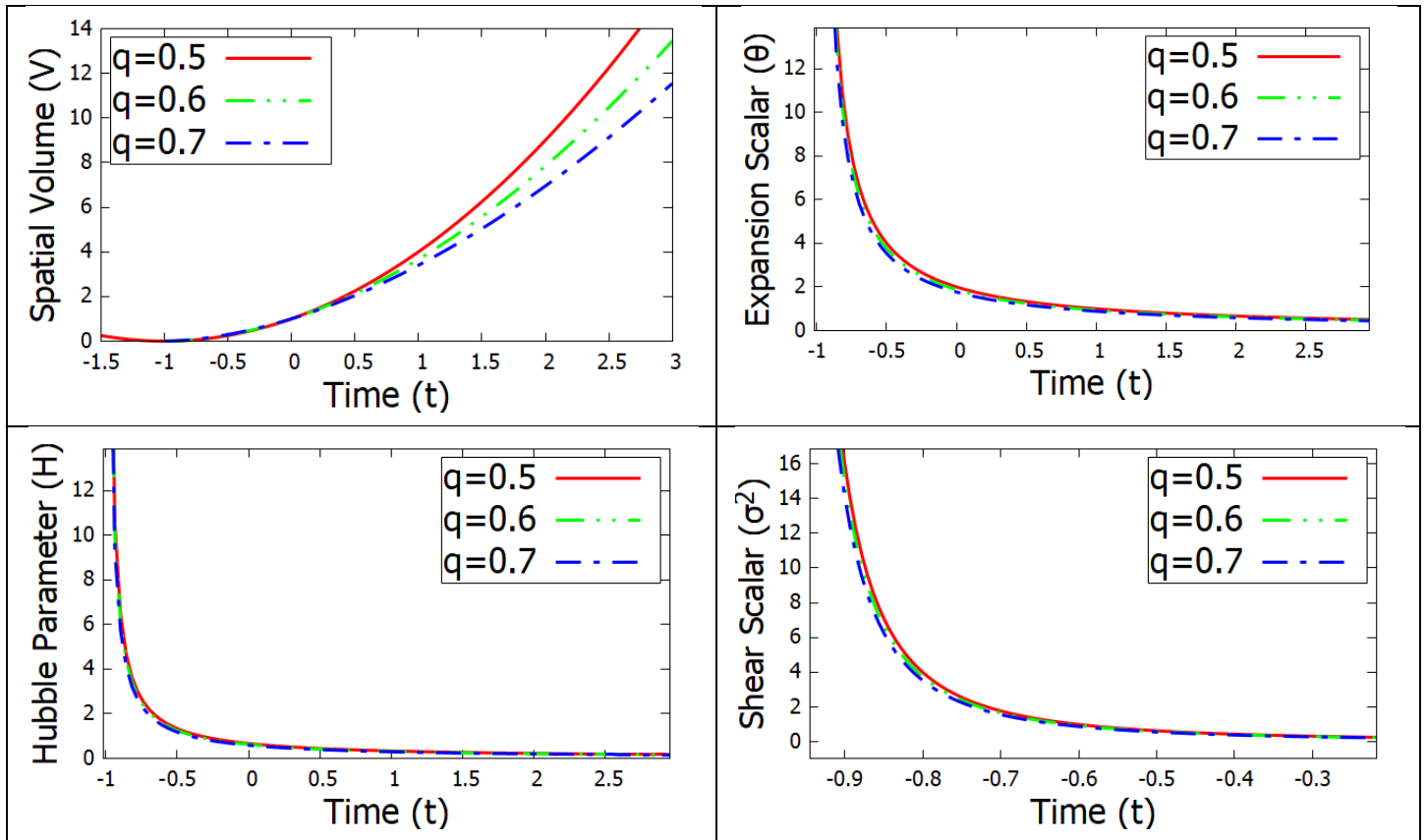


Fig.3

Also, the expression for the energy density W , energy flow vector S and stress tensor $\sigma_{\alpha\beta}$ are

Fig.3.

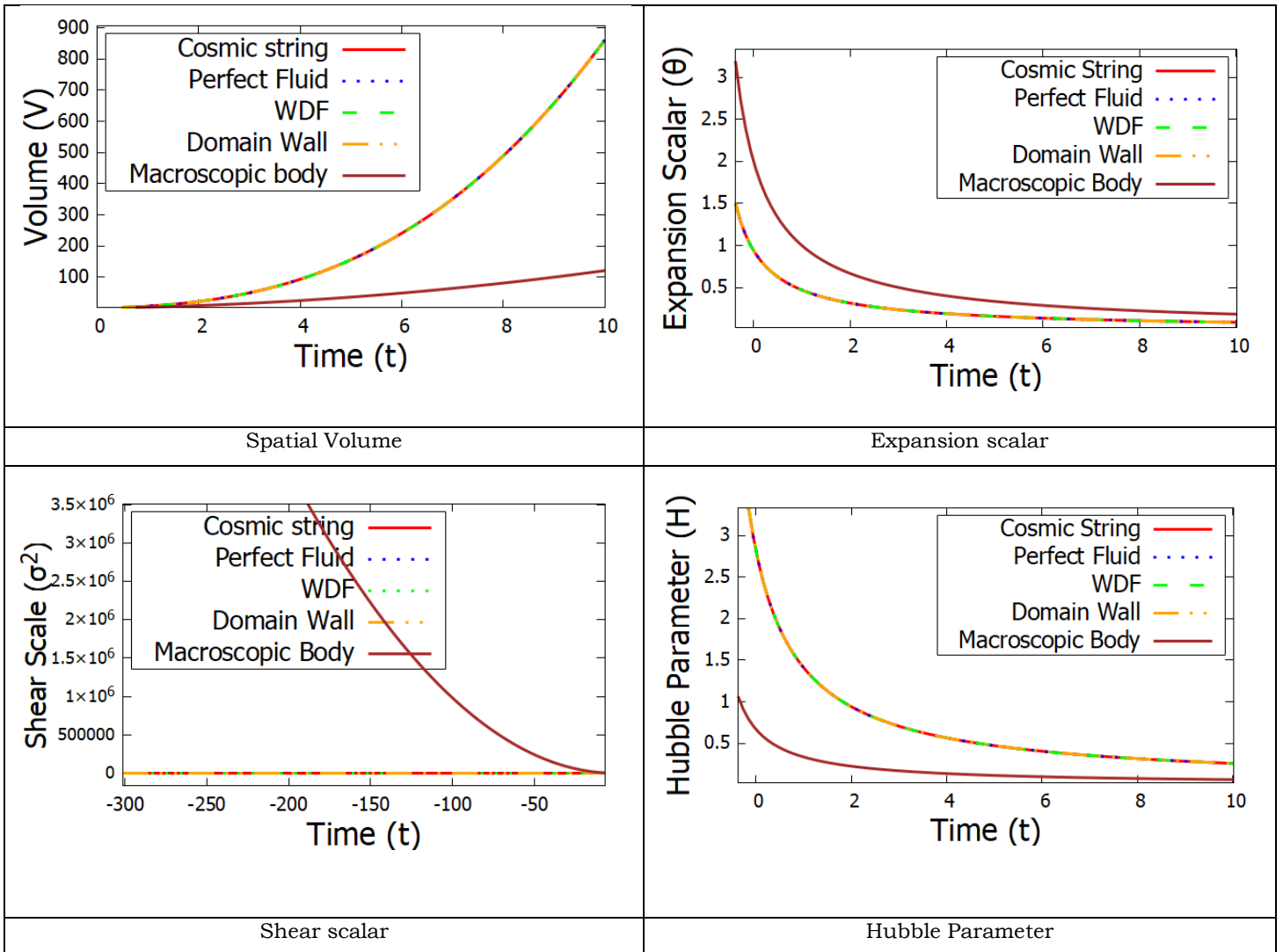
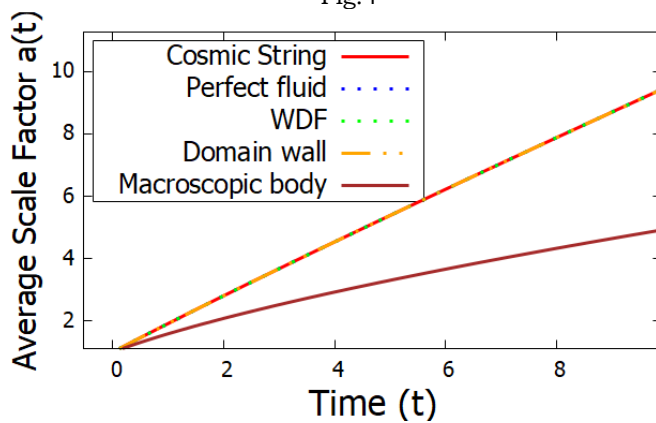


Figure 3 show that volume increases with time. Volume $V \rightarrow \infty$ as $t \rightarrow \infty$. Hubble parameter decreases with increasing time. The expansion scalar and shear scalar are decreasing with increase in time and both vanishes as $t \rightarrow \infty$.

Fig.4



Also the graph of Average Scale Factor vs. time

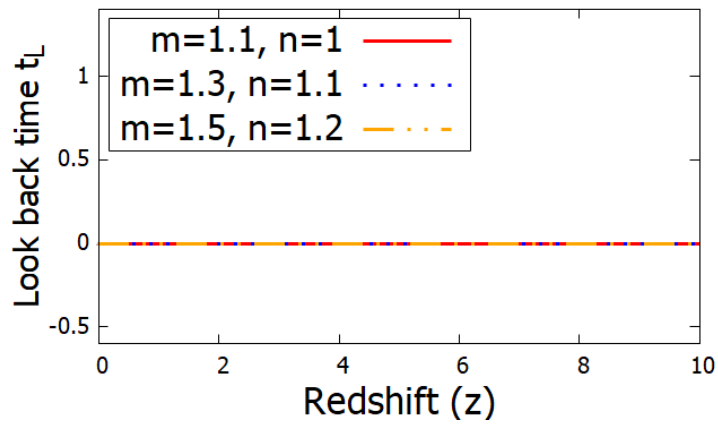


Figure 5: Plot of Look back time vs. red shift for $k_3 = 0.1, H_0 = 75$

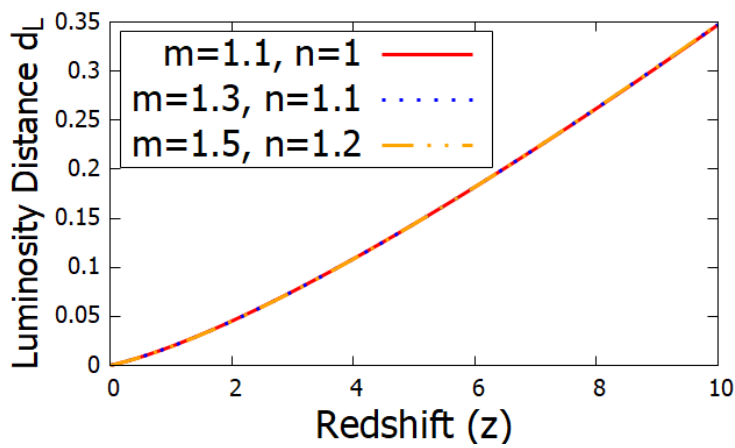


Figure 6: plot of luminosity distance vs. red shift for $k_6 = 1, H_0 = 75$

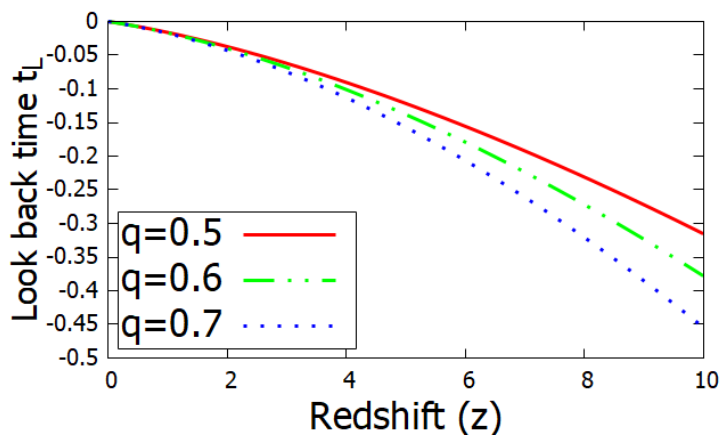


Fig.7

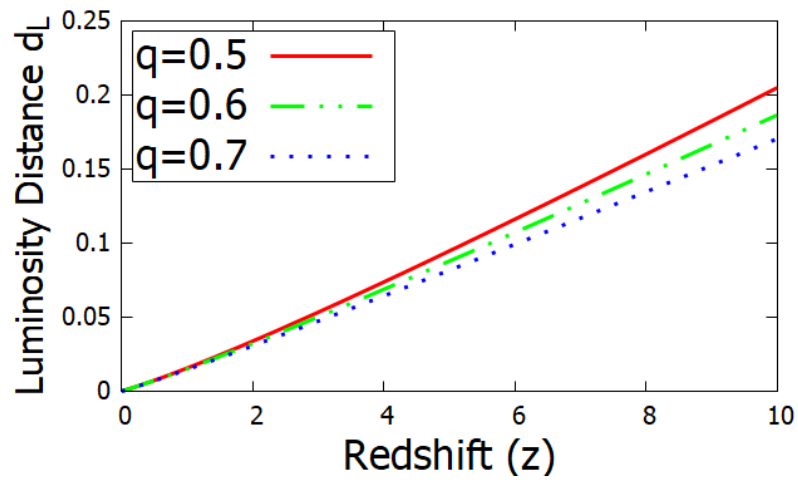


Fig.8

Above figure 8 shows that the Luminosity distance increase faster with the redshift.