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NAKEDNESS AND CURVATURE STRENGTH OF SINGULARITIES ARISING IN THE HIGHER-DIMENSIONAL SPACETIMES

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Abstract
In the present work we examine the nature of central singularity forming in the higher dimensional spherically symmetric collapse of dust cloud and it is shown that this is always a strong naked singularity where gravitational tidal forces diverge powerfully. An important consequence is that the nature of the naked singularity forming in the dust collapse turns out to be stable against the perturbations in dimension of the spacetime. Thus we have shown that the higher dimensional gravitational collapse of dust violates the cosmic censorship conjecture.
Keywords : gravitational collapse, naked singularity , block hole , cosmic censorship hypothesis.

1 Introduction

Investigations of exact solutions of the Einstein equations have shown that a large number of them contain singularities. In particular, singularities are present in those solutions that constitute reasonable models of the final stages of the evolution of stars of sufficient matter concentration. Some important questions about these singularities are: Can these singularities be observed? Whether dimensionality plays any fundamental role in the formation of naked singularities? Whether higher dimensional collapse yields a naked singularity? There is a hope, however, that we may have some control as to where the singularities may appear.

The cosmic censorship hypothesis (CCH) is an open challenging problem in general relativity [1]. It states that under the realistic conditions gravitational collapse of spacetime does not yields a naked singularity. The existence of naked singularities is still difficult to understand physically and several suggestions have been made to maintain the viability of CCH.

The principle aim of the present work is to show the existence of naked singularities in the five dimensional dust collapse. At present time, however, CCH is not yet proven, on the contrary, many counter examples candidates have been found in general relativity. The generic occurrence of naked singularities has been shown in the spherical dust collapse represented by Tolman-Bondi-Lemaitre (T-B-L) solution [2 - 5].

The T-B-L model of spherical inhomogeneous dust is the simplest model that allows the formation of both black holes and naked singularities. It is defined by specifying two function energy density function $F(r)$ and velocity distribution function $f(r)$, where r is a radial coordinate. The former represents the weighted mass contained within the matter shell labeled by r and the latter relates to the velocity profile within the collapsing cloud at the initial time.

In the present work, we would like to see the effect of extra dimension on the gravitational collapse of T-B-L spacetime. We would like to see whether five dimensional collapse of dust contradicts CCH or not? We also investigate the strength of naked singularities arising in this spacetime.

The paper is organized as follows: In Sec. 2 and 3, we discuss the nature of singularities arising in four and five-dimensional spacetime. In Sec. 3.1, we analyze the apparent horizon formation. In Sec. 3.2 and 3.3, we study the visible singularity and strength of naked singularity arising in gravitational collapse. We conclude the paper in Sec. 4.

2. Naked Singularities in Four-Dimensional Spacetime

Before discussing the higher dimensional gravitational collapse of T-B-L spacetime, we briefly mention the nature of singularity arising in 4-dimensional spacetime. The Tolman-Bondi solution represents a spherically symmetric cloud of dust collapsing under the action of its own gravity. The metric is assumed to be diagonal and spherically symmetric. The line element in comoving coordinates $(t,\ r,\ \theta, \phi)$ is given by [2-4]

$$ds^2 = -dt^2 + \frac{R'^2}{1+f}\, dr^2 + R^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{1}$$

where f is an arbitrary function of the comoving coordinate r satisfying $f > -1$. $R(t,r)$ is the physical radius at time t of the shell labeled by r. Prime denotes partial derivative with respect to r. The energy momentum tensor for above metric is given by

$$T^{ij} = \varepsilon\, \delta^i_t\, \delta^j_t, \tag{2}$$

where $\varepsilon(F,r)$ is the energy density of the cloud. The Einstein equations for this metric are

$$\varepsilon(F,r) = \frac{F'}{R^2\, R'}, \tag{3}$$

$$\ddot{R} = \frac{F}{R} + f, \tag{4}$$

(we have set up $\frac{8\pi G}{c^4} = 1$).

F is an arbitrary function of r, dot denotes partial derivative with respect to time t. Since we are concerned with collapse we assume $\dot{R} < 0$.)

The Eq.(4) can be integrated to obtain R as a function of F and r, is given by the relation

$$R^{3/2} = \frac{3}{2}\sqrt{F}\,[\,f_0(r)t\,], \tag{5}$$

$f_0(r)$ is the constant of integration which we determined by noting that there is a scaling freedom in the choice of r.

It follows from Eq.(3) that the function $F(r)$ becomes fixed once. The initial density distribution $\varepsilon(0,r) = \rho(r)$ is given i.e.

$$F(r) = \int \rho(r)\, r^2\, dr \tag{6}$$

If the initial density $\rho(r)$ has a series of expansion [6]

$$\rho(r) = \rho_0 + \rho_1 r + \frac{1}{2!}\rho_2 r^2 + \frac{1}{3!}\rho_2 r^3 + \dots \tag{7}$$

near the center $r = 0$, the resulting series expansion for the mass function $F(r)$ is

$$F(r) = F_0 r^3 + F_1 r^4 + F_2 r^5 + F_3 r^6 + \dots, \tag{8}$$

where

$$F_q = \frac{\rho_q}{q!\,(q+3)}, \quad q = 0, 1, 2, 3, \dots \tag{9}$$

As we are considering only those density functions which are decreasing away from center, first non-vanishing derivative of density should be negative.

From Eq.(5) we obtain

$$t_c(r) = \frac{2\, r^{3/2}}{3\sqrt{F(r)}}, \tag{10}$$

where $t_c(r)$ is the time at which area radius of the shell r become zero.
From above equation we obtain

$$t_0 = \frac{2}{3\sqrt{F_0}}\ , \tag{11}$$

where t_0 is the time at which central singularity forms.

For the marginally bound (i.e. $f = 0$) collapse it has been shown that [7]

i) If $\rho_1 < 0,$ the singularity is naked and weak.

ii) If $\rho_1 = 0, \quad \rho_2 < 0$, the singularity is naked and weak.

iii) If $\rho_1 = 0, \quad \rho_2 = 0, \quad \rho_3 < 0$, the singularity is naked, if $\xi = \frac{\sqrt{3}}{4}\frac{\rho_3}{\rho_0^{5k}}$ is less than critical value $\xi_c = -25.9904$ and covered if $\xi > \xi_c$. Further, the naked singularity is a strong curvature singularity.

iv) If $\rho_1 = \rho_2 = \rho_3 = 0\ ,$ the singularity is not naked i.e. the collapse ends into a black hole.

3. Higher Dimensional Tolman-Bondi-Lemaitre Spacetime

The spherically symmetric inhomogeneous dust cloud in five-dimensional spacetime [8,9,10] is given by

$$ds^2 = -dt^2 + \frac{R'^2}{1+f}\ dr^2 + R^2\left(d\theta_1{}^2 + \sin^2\theta_1\, d\theta_2{}^2 + \sin^2\theta_1 \sin^2\theta_2\, d\theta_3{}^2\right). \tag{12}$$

where $f(r)$ is arbitrary function of comoving coordinate r, satisfying $f > -1$. $R(t,r)$ is the physical radius at time t of the shell labeled by r in the sense that $4\pi R^2(r,t)$ is the proper area of shell at time t.

Let

$$\{x^\mu\} = \{t, r, \theta_1, \theta_2, \theta_3\}, \qquad (\mu = 0, 1, 2, 3, 4).$$

The non-vanishing metric components are

$$g^{00} = -1, \qquad g^{11} = \frac{1+f}{R'^2},$$
$$g^{22} = \frac{1}{R^2}, \qquad g^{33} = \frac{1}{R^2 \sin^2\theta_1}, \tag{13}$$
$$g^{44} = \frac{1}{R^2 \sin^2\theta_1 \sin^2\theta_2}\ .$$

The non-vanishing christoffel symbols associated with metric (12) are

$$\Gamma^{0}_{11} = \frac{R'\dot{R}'}{1+f}, \qquad \Gamma^{0}_{22} = R\dot{R},$$
$$\Gamma^{0}_{33} = R\dot{R}\sin^{2}\theta_{1}, \qquad \Gamma^{0}_{44} = R\dot{R}\sin^{2}\theta_{1}\sin^{2}\theta_{2},$$
$$\Gamma^{1}_{10} = \frac{\dot{R}'}{R'}, \qquad \Gamma^{1}_{11} = \frac{R''}{R'} - \frac{f'}{2(1+f)},$$
$$\Gamma^{1}_{22} = \frac{-R(1+f)}{R'}, \qquad \Gamma^{1}_{33} = \frac{-R}{R'}\sin^{2}\theta_{1}(1+f),$$
$$\Gamma^{1}_{44} = \frac{-R}{R'}\sin^{2}\theta_{1}\sin^{2}\theta_{2}\,(1+f), \qquad \Gamma^{2}_{12} = \frac{R'}{R},$$
$$\Gamma^{2}_{20} = \frac{\dot{R}}{R}, \qquad \Gamma^{2}_{33} = \sin\theta_{1}\cos\theta_{1},$$
$$\Gamma^{2}_{44} = -\sin\theta_{1}\cos\theta_{1}\sin^{2}\theta_{2}, \quad \Gamma^{3}_{03} = \frac{\dot{R}}{R}, \tag{14}$$
$$\Gamma^{3}_{13} = \frac{R'}{R}, \qquad \Gamma^{3}_{23} = \cot\theta_{1},$$
$$\Gamma^{3}_{44} = -\sin\theta_{2}\cos\theta_{2}, \qquad \Gamma^{4}_{04} = \frac{\dot{R}}{R},$$
$$\Gamma^{4}_{14} = \frac{R'}{R}, \qquad \Gamma^{4}_{24} = \cot\theta_{1},$$
$$\Gamma^{4}_{34} = \cot\theta_{2}.$$

Ricci tensors for the metric (12) are

$$R_{00} = \frac{-\ddot{R}}{R'} - \frac{3\ddot{R}}{R}, \qquad R_{01} = 0.$$

$$R_{11} = \frac{R'\ddot{R}'}{1+f} + \frac{3\dot{R}R'\dot{R}'}{R(1+f)} - \frac{R'f}{1+f}. \tag{15}$$

The energy momentum tensor is given by

$$T^{ij} = \varepsilon\,\delta^{i}_{t}\,\delta^{j}_{t}, \tag{16}$$

where

$$\varepsilon(F,r) = \frac{3F'}{2R^{3}R'}, \tag{17}$$

we assume energy density $\varepsilon(f,r)$ such that it is higher at the center and decreasing away from the center. The function $R(r,t)$ is given by

$$\dot{R}^{2} = \frac{F(r)}{R^{2}} + f(r). \tag{18}$$

Since we are considering the collapsing model, it requires $\dot{R} < 0$.
We use the rescaling freedom in r to set

$$R(0,r) = r, \tag{19}$$

so that the physical area radius R increases monotonically in r and $R' = 1$. Also there will be no shell-crossing singularities on the initial surface. It has been shown in Ref.[2] that shell-crossing singularities are gravitationally weak, through which the spacetime may sometimes be extended. Hence we shall concentrate our attentions on only shell focusing singularities $(\text{i.e. } R = 0)$.

The central singularity at $r = 0$, where density and curvature are infinite, is naked if there are outgoing non-space like geodesics which reach far away observer in the future and terminate at the singularity in the past. Integration of Eq.(18) yields the solution

$$t - t_s(r) = \frac{-R^2}{\sqrt{F}} G\left(\frac{f R^2}{F}\right), \tag{20}$$

where $G(y)$ is a strictly real positive and bounded function given by

$$G(y) = \sqrt{\frac{1+y}{y}}, \qquad y \neq 0$$

$$= \frac{1}{2}, \qquad y = 0 \tag{21}$$

Using scaling freedom $R(0,r) = r,$ Eq.(2.19) gives

$$t_s(r) = \frac{r^2}{\sqrt{F}} G\left(\frac{r^2 f}{F}\right),$$

where $t_s(r)$ gives the time at which the physical radius R becomes zero , hence ranges for t and r are given by

$$-\infty < t \leq t_s(r) \quad \text{and} \quad 0 \leq r < \infty. \tag{22}$$

Since the shell-crossing singularities $(i.e.\ R' = 0\ ,\ R > 0)$ are gravitational weak [2] we consider only the shell focusing singularities $(R = 0)$. From Eqs.(16)-(21) , we can obtain

$$R' = r^{\alpha-1}\left[\frac{1}{2}(\eta-\beta)X + \left(\Theta - \left(\frac{\eta}{2}-\beta\right)X^2\, G(P X^2)\right)\left(P + \frac{1}{X^2}\right)^{1/2}\right]$$
$$= r^{\alpha-1} H(X,r), \tag{23}$$

where we have used the following notations

$$u = r^{\alpha}, \qquad X = \frac{R}{u}, \qquad \eta(r) = \frac{r f'}{F}, \qquad \beta(r) = \frac{r f'}{F},$$

$$p(r) = \frac{r^2 f}{F}, \quad \Lambda = \frac{\sqrt{F}}{u}, \quad P = \frac{f u^2}{F}, \tag{24}$$

$$\Theta = \frac{1}{r^{2(\alpha-1)}}\left(G(p)\left(\frac{\eta}{2} - \beta\right) + \frac{2 + \beta - \eta}{2\sqrt{1+p}} \right),$$

$$H(X,R) = \frac{1}{2}(\eta - \beta)X + \left(\Theta - \left(\frac{\eta}{2} - \beta\right) X^2\, G(P X^2) \right)\left(P + \frac{1}{X^2} \right)^{1/2}. \tag{25}$$

For marginally bound collapse $(\text{i.e. } f = 0)$, $B(r)$ will be zero.

The parameter α (which satisfies $\alpha \geq 1$) is introduced here for examining the structure of the central singularity at $r = 0$.

Kretschman scalar $(K = R_{abcd}\, R^{abcd})$ for the metric (12) is given by

$$K = \frac{A F'^2}{R^6 R'^2} + \frac{B F F'}{R^7 R'} + \frac{C F^2}{R^8}, \tag{26}$$

where A,B and C are some constants.

It can be seen from above equation that the shell focusing singularities occurring in T-B-L spacetime are also scalar polynomial singularity as Kretschman scalar also vanishes at $R = 0$.

To investigate the structure of the collapse we need to consider the radial null geodesics defined by $ds^2 = 0$ taking $\dot{\phi} = \dot{\theta} = 0$ into account.

In order to determine whether or not the singularity is naked, we investigate the future-directed outgoing null geodesics emanating from the singularity. We want to determine if such a singularity is naked i.e. if there exists at least one future-directed radial null geodesics with past end point at the singularity.
Equation for null geodesics are given by

$$K^t = \frac{dt}{dk} = \frac{P}{R}, \tag{27}$$

$$K^r = \frac{dr}{dk} = \frac{K^t \sqrt{1+f}}{R'} = \frac{P\sqrt{1+f}}{R R'}, \tag{28}$$

where the function $P(t,r)$ satisfies the equation

$$\frac{dp}{dk} + P^2\left(\frac{\dot{R}'}{R' R} - \frac{\dot{R}}{R^2} - \frac{\sqrt{1+f}}{R^2} \right) = 0 . \tag{29}$$

Let $u = r^{\alpha} \quad (\alpha \geq 1)$ then

$$\frac{dR}{du} = \frac{1}{\alpha\, r^{\alpha-1}} \left(R' + \dot{R}\, \frac{dt}{dr} \right). \tag{30}$$

From Eq.(12) we can observe that for outgoing radial null geodesics

$$\frac{dt}{dr} = \frac{R'}{\sqrt{1+f}}. \tag{31}$$

From Eq.(18) we have

$$\dot{R} = \pm \sqrt{\frac{F}{R^2} + f}, \tag{32}$$

where the plus or minus sign corresponds to expansion or collapse respectively. If $\dot{R} < 0$ then every dust shell implodes and inevitably collapses to vanishing proper area in a proper time. Since we are considering the collapsing case, we should consider negative sign. Hence we must have

$$\dot{R} = -\sqrt{\frac{F}{R^2} + f}. \tag{33}$$

With the help of Eqs.(31) and (33) , we can write the Eq.(30) as

$$\frac{dR}{du} = \frac{R'}{\alpha\, r^{\alpha-1}} \left(1 + \frac{\sqrt{\frac{F}{R^2} + f}}{\sqrt{1 + f}} \right) \tag{34}$$

$$= \frac{H(X,u)}{\alpha} \left(1 - \frac{\sqrt{\frac{\Lambda}{X^2} + f}}{\sqrt{1 + f}} \right) \tag{35}$$

$$= U(X,u) \tag{36}$$

It is to be noted that $H(X,u)$ appeared in Eq.(35) is strictly positive and non-zero for all $r > 0$.

3.1 Apparent Horizon

No study on gravitational collapse is complete without the discussion of apparent horizon. Spherical symmetry implies that the apparent horizon will only depend on the radial and time coordinates. Therefore, we begin by finding the differential equation governing the class of all radial null geodesics, which is done by putting $ds^2 = d\theta = d\phi = 0$. If the formation of the horizon precedes the formation of the central singularity then the singularity will be necessarily covered i.e. collapse will convert into black hole. On the other hand, if horizon formation occurs after the singularity formation, there may be future-directed non-spacelike geodesics that end in the past at the singularity. Then the final end state would be a naked singularity.

When a large amount of mass is contained in a small region of spacetime, a trapped surface forms around it. Therefore as the matter collapses under the influence of a gravitational force, there is a possibility that a trapped surface will form as the collapse proceeds. If this happens then on a sufficiently late time special surface, there will be a boundary that separates the trapped region from the normal region. This boundary is known as *apparent horizon.*

If the null geodesic terminates in the past at the singularity with a definite tangent, then at the singularity, the tangent to the geodesic dR/du is positive and must have a finite value. From Eq.(34) it can be observed that dR/du is positive if

$$1-\frac{\sqrt{\frac{F}{R^2}+f}}{\sqrt{1+f}} > 0 , \tag{37}$$

i.e. if

$$\sqrt{1+f} > \sqrt{\frac{F}{R^2}+f} . \tag{38}$$

Since both sides are positive, we can write

$$1+f > \frac{F}{R^2} + f ,$$

which in turns reduces to $R^2 > F$.

Thus the boundary of the trapped surface in the five-dimensional T-B-L spacetimes is given by

$$R = \sqrt{F} . \tag{39}$$

Using above value of $R,$ Eq.(19) becomes

$$t_{ah}(r) = t_s(r) - \sqrt{F}\, G(f) , \tag{40}$$

where $t_{ah}(r)$ denotes the time at which apparent horizon forms.

It can be easily seen from the above equation that all the points on the singularity curve $t_s(r)$, other than the central point $(r=0)$ are covered by the apparent horizon. This is because, both the functions $F(r)$ and $G(r)$ are strictly positive for $(r>0)$, with $F(r) = 0$ at $r=0$.Therefore for all $r>0$,

$$t_s(r) > t_{ah}(r) \text{ and } t_s(0) = t_{ah}(0) . \tag{41}$$

Above equation shows that non central singularities always form later than apparent horizon, hence they are covered and can not be naked. But at $r = 0$, time of formation of singularity and apparent horizon is exactly same; hence there is chance to some radial null geodesic to escape from the singularity. Thus only central singularity could be naked while non central singularities are covered.

The singularity is naked if and only if there exist an outgoing null geodesic which emanates from the singularity.

The radial null rays are given by

$$X_0 = \lim_{\substack{R\to 0\\ r\to 0}} \frac{R}{r^{\alpha}} = \lim_{\substack{R\to 0\\ r\to 0}} \frac{R}{u} \tag{42}$$

$$= \lim_{\substack{R\to 0\\ r\to 0}} \frac{dR}{du} . \tag{43}$$

The variable X can be interpreted as the tangent to the outgoing geodesic, in $R,\ u$ plane. It can be shown that if the equation

$$V(X) = U(X,0) - X$$

$$= \frac{H(X,0)}{\alpha}\left(1 + \frac{\sqrt{\frac{\Lambda_0{}^2}{X^2} + f_0}}{\sqrt{1 + f_0}}\right) - X$$

$$= 0 \tag{44}$$

admits a real positive root, then the central singularity at with $r = 0, \quad R = 0$ is naked. If the Eq.(44) does not have real and positive root , then collapse will convert into a black hole.

3.2 Visible Singularity

Consider $f(r)$ and $F(r)$ as

$$f(r) = f_0 r^2 \left(1 + f_1 r^2\right), \tag{45}$$

$$F(r) = F_0 r^4 , \tag{46}$$

$$\frac{f_0}{F_0} = P_0 > -1 , \tag{47}$$

where $f_0,\ F_0$ and f_1 are constants.

Therefore we can obtain

$$\beta_0 = 2 \ , \ \eta(r) = \mu \ , \ p(r) = p_0\left(1 + f_1 r^2\right) \ , \ \alpha = 2 ,$$

$$\Theta_0 = f_1\left[\frac{1}{\sqrt{1+p_0}} - 2G(p_0)\right], \quad \Lambda(r) = \sqrt{F_0} . \tag{48}$$

$$H(X,0) = X + \frac{\Theta_0}{X} , \tag{49}$$

Therefore Eq.(44) becomes

$$V(X) = \frac{1}{2}\left[1 - \frac{\Lambda_0}{X}\right]\left[X + \frac{\Theta_0}{X}\right] - X = 0 , \tag{50}$$

i.e.

$$X^3 + \sqrt{F_0}\ X^2 - \Theta_0 X + \Theta_0\sqrt{F_0} = 0 \tag{51}$$

Some numerical computations show that Eq.(2.51) has positive real root if

$$\frac{F_0}{\Theta_0} < \frac{-11 + 5\sqrt{5}}{2} . \tag{52}$$

Thus if the above equality is satisfied then the collapse could convert in to a naked singularity.

3.3 Strength of a Naked Singularity

We now investigate the strength of a naked singularity. The main importance of determining the strength of a singularity is due to the fact that the CCH does not need to rule out the possibility of the occurrence of the weak naked singularity [11].

A singularity is said to be strong if the collapsing objects do get crushed to a zero volume at the singularity and a weak one if they do not.

According to Clarke and Krolak [12] criteria, the singularity is said to be strong in sense of Tipler [13] if at least along one radial null geodesic, we must have

$$\lim_{k \to 0} k^2 \Psi = \lim_{k \to 0} k^2 R_{ab} K^a K^b > 0 , \quad (53)$$

where K^a is the tangent to the null geodesics and R_{ab} is the Ricci tensor.

Thus

$$\lim_{k \to 0} k^2 R_{ab} K^a K^b = \lim_{k \to 0} k^2 \frac{3F'}{2R^3 R'} \left(K^t\right)^2$$

$$= \frac{3\eta_0 \Lambda_0{}^2}{2\alpha X_0{}^6} \lim_{k \to 0} \left(\frac{k P}{r^{2\varepsilon}}\right), \quad (54)$$

where

$$K^t = \frac{P}{R} , \quad (55)$$

$$K^r = \frac{\sqrt{1+f}}{R'} K^t , \quad (56)$$

where P satisfies the differential Eq.(29)

Using L-Hospital's rule and Eqs.(4) to (10), (55) and (56) and the fact that at the singularity $r \to 0, \; X \to X_0$, we have

$$\lim_{k \to 0} k^2 R_{ab} K^a K^b > 0 .$$

Thus five-dimensional gravitational collapse satisfied the condition of Clarke and Krolak which shows that naked singularities arising in this gravitational collapse are gravitationally strong.

4. Conclusion

A naked singularity is a singularity which is visible to a far away observer, i.e. outgoing light rays starting from the singularity terminate on the singularity in the past. Tolman-Bondi spacetime has been extensively used to study the naked singularities [2-5]. We have extended this study to higher dimensional Tolman-Bondi metric and found that strong curvature naked singularities do arise in these spacetimes.

We have also shown that dimensionality of spacetime does not essentially change the basic nature of the singularity of an inhomogeneous dust collapse. In other word, we can argue that the naked singularities found in Tolman-Bondi dust collapse are stable with respect to perturbations in dimensions of the spacetime. Since we found naked singularities in higher-dimensional dust collapse, higher dimensional spacetimes violate the cosmic censorship conjecture.

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